A Ramsey Version of Graph Saturation

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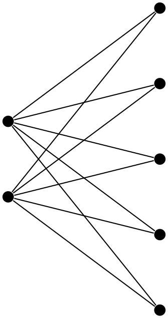

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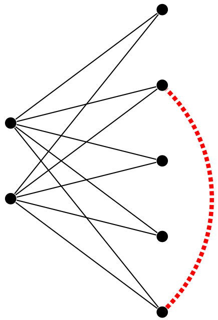

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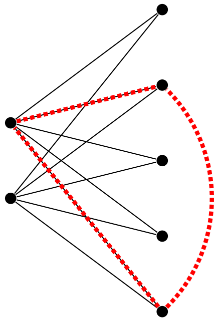

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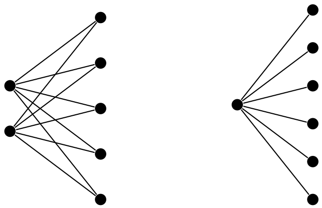

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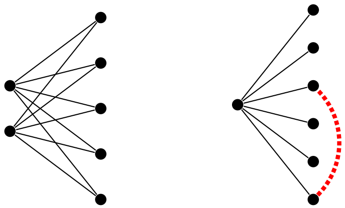

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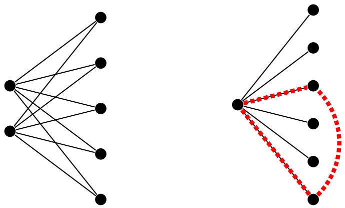

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Given a forbidden family of graphs $\mathcal{F}$, a graph G is $\mathcal{F}$-**saturated** if no member of $\mathcal{F}$ is a subgraph of G, but for every $e \in \overline{G}$, some member of $\mathcal{F}$ is a subgraph of $G + e$.
The **saturation number** $\mathbf{sat}(n;\mathcal{F})$ is the smallest number of edges over all n-vertex graphs that are $\mathcal{F}$-saturated.

Ramsey-Minimal Families

Definitions

A graph G is **$(H_1, \ldots, H_k)$-Ramsey minimal** if $G \to (H_1, \ldots, H_k)$ but for any $e \in E(G)$, $G - e \not\to (H_1, \ldots, H_k)$.

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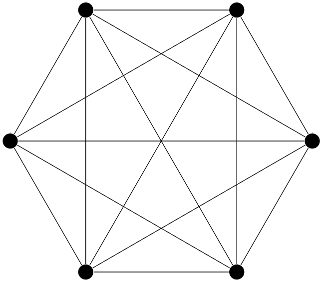

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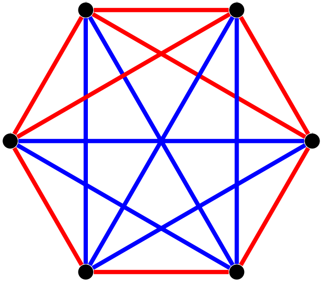

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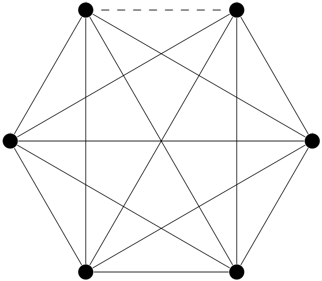

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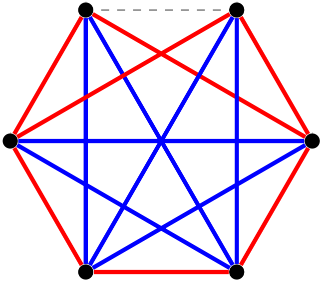

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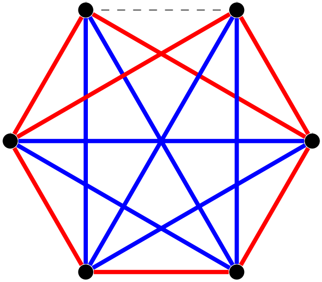

Definitions

$\mathcal{R}_{\mathbf{min}}(\mathbf{H_1}, \ldots, \mathbf{H_k}) = \mathcal{R}_{min} = \{G : G \text{ is } (H_1, \ldots, H_k)\text{-Ramsey minimal}\}$

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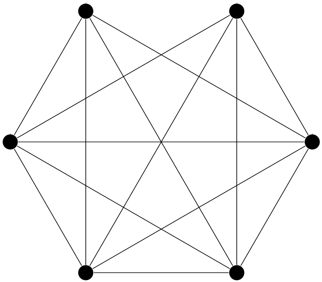

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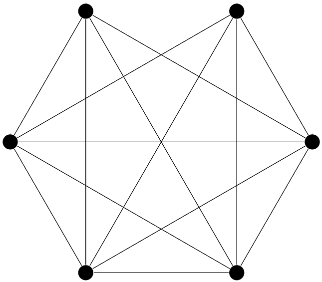

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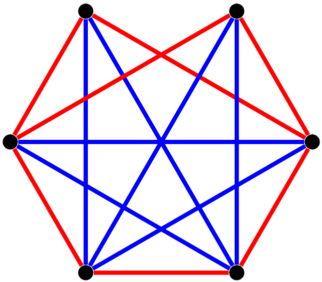

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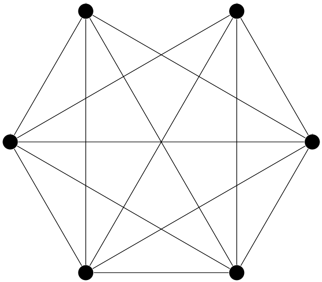

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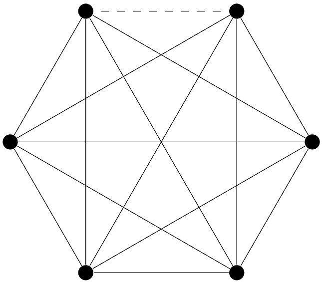

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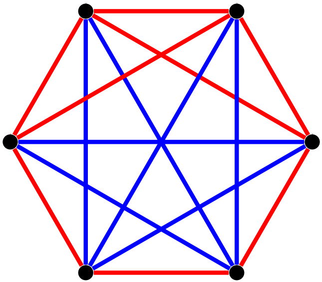

Saturation of $\mathcal{R}_{min}(K_{k_1}, \ldots, K_{k_t})$

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$$K_{r-2} \vee \overline{K_s}$$

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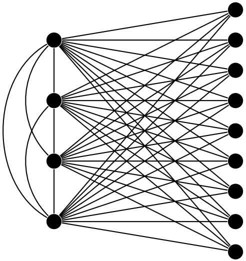

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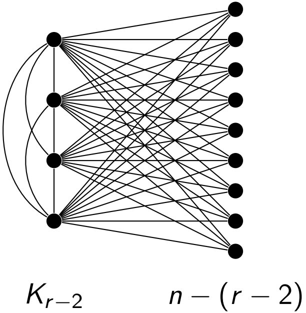

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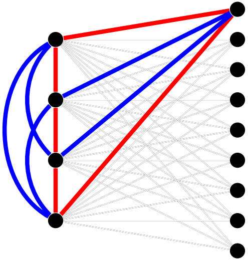

K_{r-2} $n - (r - 2)$

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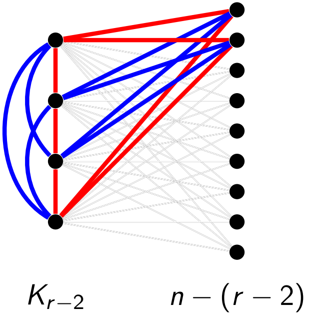

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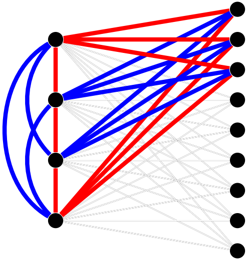

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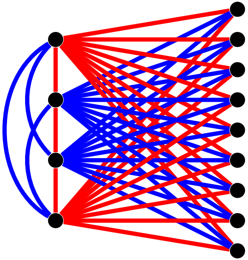

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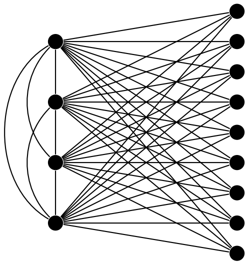

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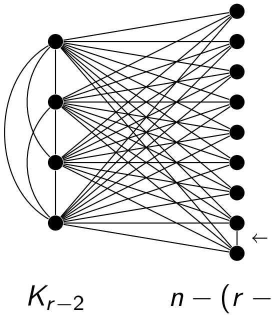

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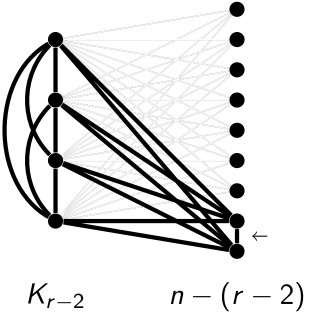

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Hanson-Toft Conjecture, 1987

$$sat(n; \mathcal{R}_{min}(K_{k_1}, \ldots, K_{k_t})) = \begin{cases} \binom{n}{2} & n < r \\ \binom{r-2}{2} + (r-2)(n-r+2) & n \geq r \end{cases}$$

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Chen, Ferrara, Gould, Magnant, Schmitt; 2011

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Matchings

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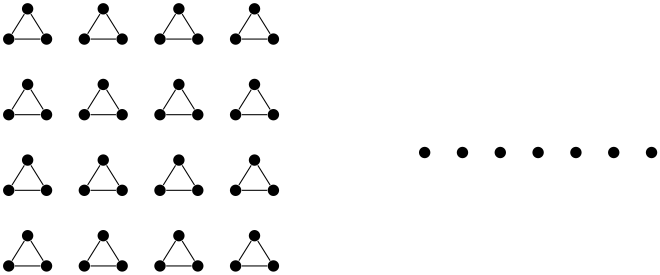

$(5K_2, 5K_2, 5K_2, 5K_2)$

Matchings

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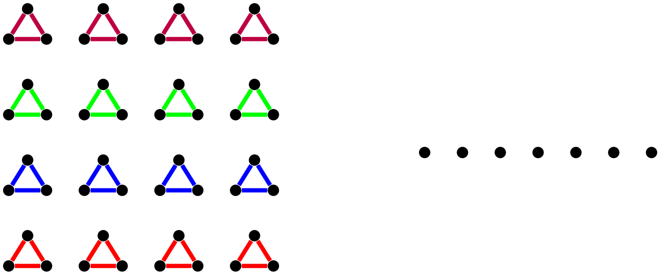

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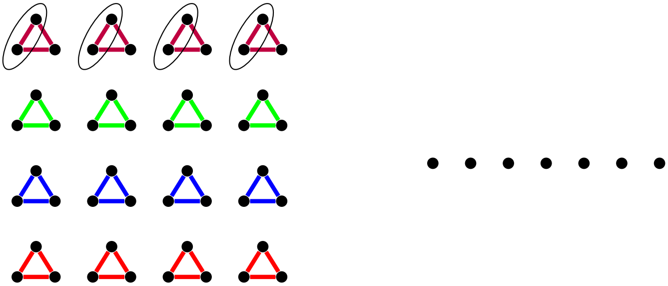

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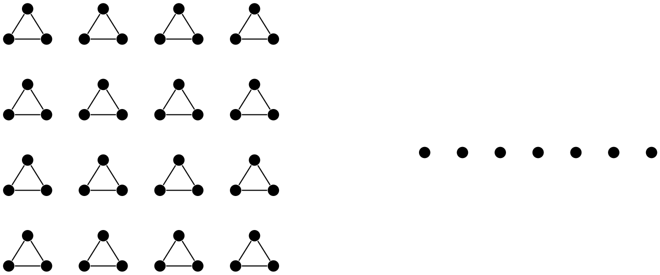

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Useful Observation: "Iterated Recoloring"

Ferrara, Kim, Y.; 2014

Looking (cleverly) at color i allows us to use results from graph saturation of the forbidden subgraph H_i.

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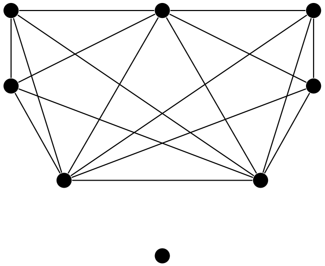

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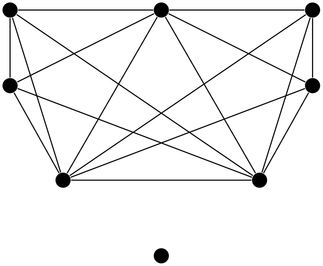

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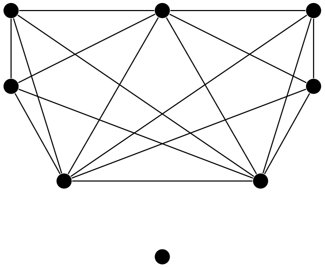

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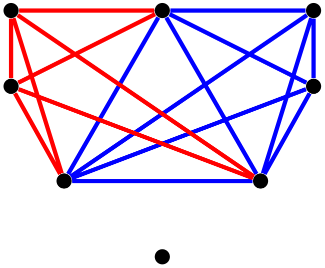

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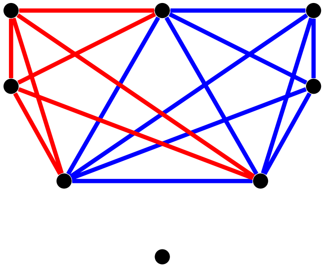

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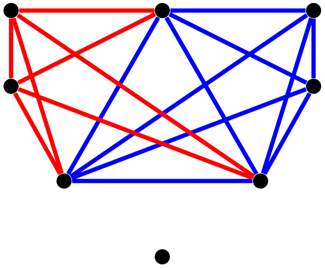

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make red-heavy

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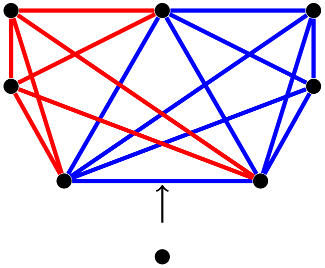

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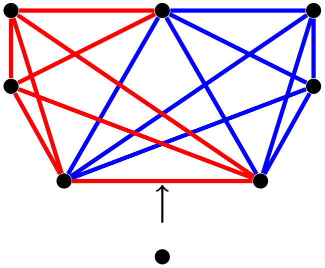

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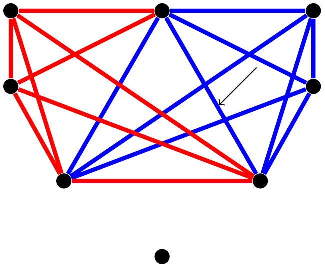

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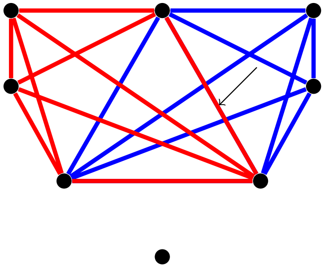

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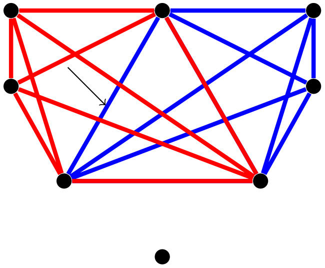

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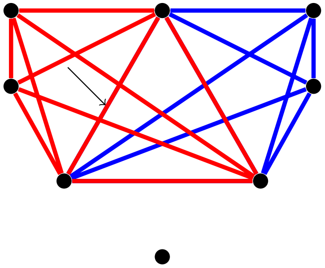

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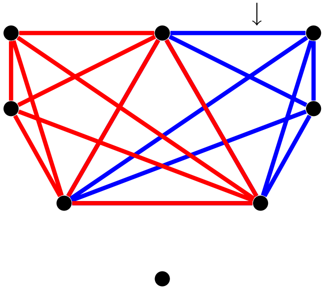

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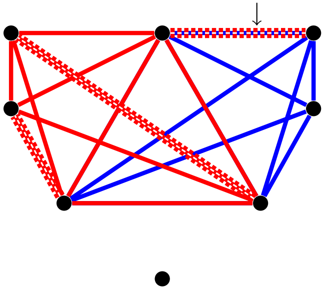

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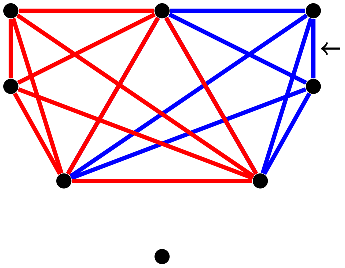

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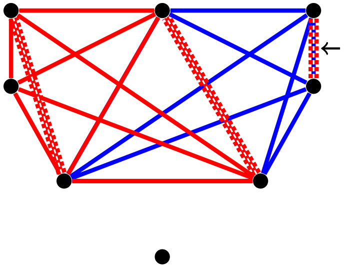

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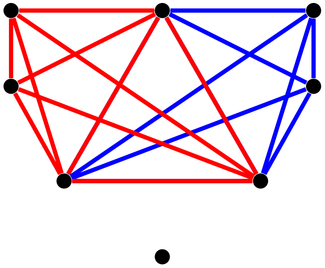

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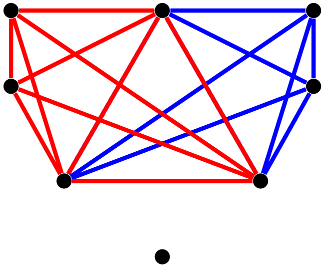

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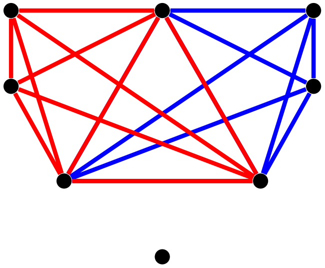

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take red subgraph

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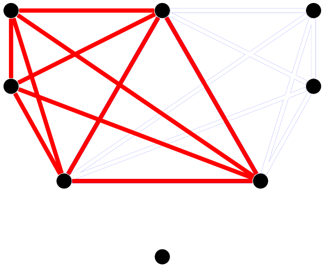

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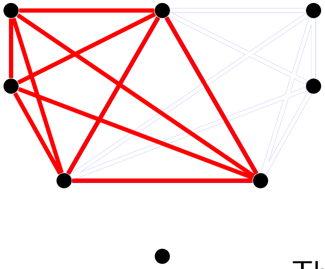

good coloring

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This (uncolored) subgraph is $3K_2$-saturated.

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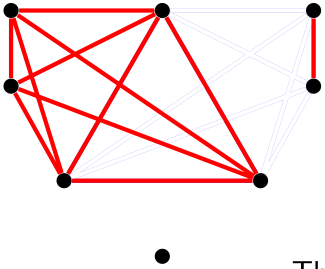

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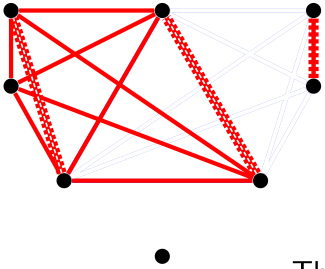

good coloring

$\downarrow$

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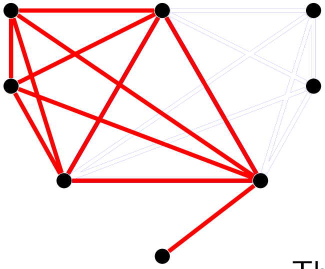

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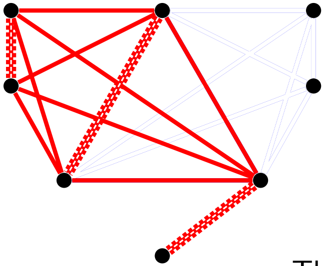

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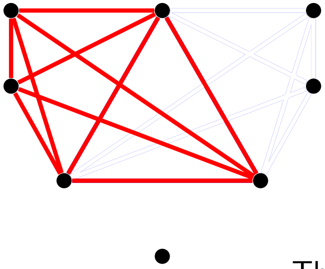

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Thanks for Listening!

- G. Chen, M. Ferrara, R. Gould, C. Magnant, J. Schmitt, **Saturation numbers for families of Ramsey-minimal graphs**, J. Combin. 2 (2011) 435-455.
- M. Ferrara, J. Kim, E. Yeager, **Ramsey-minimal saturation numbers for matchings**, Discrete Math. 322 (2014) 26-30.
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