

## Math 199 Worksheet · 17 Sept 2014

Group names:

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**Question 1.** Suppose at the start of the zombie apocalypse, every zombie lives (well, not *lives*, but you know, keeps moving) for one month, and in that month makes 5 new zombies.

- (a) Let  $a_n$  be the number of people who turned into zombies in the  $n$ th month. Suppose  $a_1 = 5$ : that is, some sole original zombie escapes from a lab and bites 5 people. Give a formula for  $a_n$ .
- (b) What is the name for this type of sequence?
- (c) This type of sequence is often used to model disease outbreaks. Why is this a reasonable model? Why is it an unreasonable model?

**Question 2.** Administrators and faculty at the University of University Town get percentage raises each year. Support staff get flat raises each year. Professors in the math department get a 2% raise each year, and custodians get \$0.50 per hour more each year. When the university starts operation this year, both professors and custodians are making \$20 per hour.

- (a) Next year, who make more money per hour, professors or custodians?
- (b) In two years, who makes more?
- (c) Give the formula for a professor's salary.
- (d) Professors will reach \$40 per hour after about 35 years. When will custodians?
- (e) In 100 years, professors will make about \$145. What about custodians?
- (f) In 200 years, professors will make about \$1059. What about custodians?
- (g) Describe the relative growth of these arithmetic and geometric sequences.

**Question 3.** At Mom's Friendly Robot Company, every employee's hourly wage changes every year either by a percentage increase (like the professors at the University of University Town), or by a flat increase (like the UUT custodians).

- (i) For each of the employees below, determine the formula  $a_n$  that describes their pay rate in year  $n$ .
- (ii) Determine whether  $a_n$  is geometric or arithmetic, give the common ratio or difference, and find  $\lim_{n \rightarrow \infty} a_n$ .
  - (a) Alice gets paid  $1/10$  robot dollars per hour in her first year,  $2/5$  robot dollars per hour her second year, and  $8/5$  robot dollars per hour her third year. (In other words,  $a_1 = 1/10$ ,  $a_2 = 2/5$ ,  $a_3 = 8/5$ .)
  - (b) For Bob,  $a_1 = 1/10$ ,  $a_2 = 2/5$ , and  $a_6 = 8/5$ .
  - (c) For Charlie,  $a_1 = 3$ ,  $a_2 = 2$ , and  $a_5 = 16/27$ .

**Question 4.** Assume that the pattern continues for the sequence below. Give the generating function and the limit. Is the sequence arithmetic or geometric?  $\frac{1}{6}, \frac{3}{18}, \frac{5}{54}, \frac{7}{162}, \frac{9}{486}, \dots$

**Question 5.** Find the limit of the sequences below.

(a)  $a_n = \frac{3n^2 + 2n - 15}{32 + 15n^2 - 9n}$

(b)  $c_n = 2 \left( \frac{2}{n^2} + 3 \right) \left( \frac{n+1}{n} \right)$

(c)  $c_n = \frac{k^n}{2}$

**Question 6.** In the peer-reviewed journal *Eurosurveillance*, Volume 19, Issue 36, 11 Sept 2014, there is an article titled EARLY TRANSMISSION DYNAMICS OF EBOLA VIRUS DISEASE (EVT), WEST AFRICA, MARCH TO AUGUST 2014. Authors H. Nishiura and G. Chowell try to determine how fast the current Ebola outbreak is spreading.

- (a) On Sept 14, Wired.com published an article about the Nishiura-Chowell paper by Maryn McKenna titled *The Mathematics of Ebola Trigger Stark Warnings: Act Now or Regret It* (available on the web at <http://www.wired.com/2014/09/r0-ebola>). In it, McKenna wrote the following:

The most arresting is a piece published last week in the journal *Eurosurveillance*, which is the peer-reviewed publication of the European Centre for Disease Prevention and Control (the EUs Stockholm-based version of the US CDC). The piece is an attempt to assess mathematically how the epidemic is growing, by using case reports to determine the reproductive number. (Note for non-epidemiology geeks: The basic reproductive number - usually shorted to R0 or R-nought - expresses how many cases of disease are likely to be caused by any one infected person. An R0 of less than 1 means an outbreak will die out; an R0 of more than 1 means an outbreak can be expected to increase. If you saw the movie *Contagion*, this is what Kate Winslet stood up and wrote on a whiteboard early in the film.)

The *Eurosurveillance* paper, by two researchers from the University of Tokyo and Arizona State University, attempts to derive what the reproductive rate has been in Guinea, Liberia and Sierra Leone. (Note for actual epidemiology geeks: The calculation is for the effective reproductive number, pegged to a point in time, hence actually  $R_t$ .) They come up with an R of at least 1, and in some cases 2; that is, at certain points, sick persons have caused disease in two others.

In terms of what you've learned in this class, explain the sentence: "An R0 of less than 1 means an outbreak will die out; an R0 of more than 1 means an outbreak can be expected to increase."

- (b) In the Nishiura-Chowell paper, the abstract reads as follows:

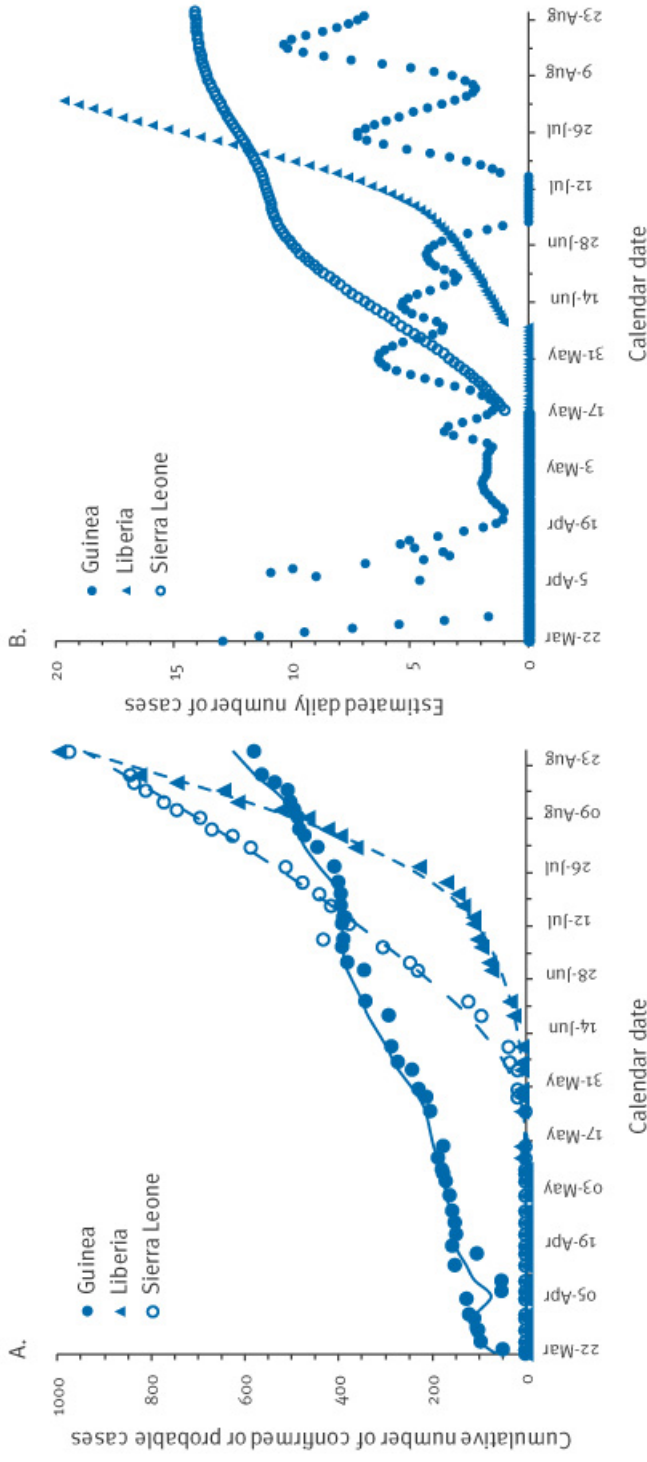
The effective reproduction number,  $R_t$ , of Ebola virus disease was estimated using country-specific data reported from Guinea, Liberia and Sierra Leone to the World Health Organization from March to August, 2014.  $R_t$  for the three countries lies consistently above 1.0 since June 2014. Country-specific  $R_t$  for Liberia and Sierra Leone have lied between 1.0 and 2.0.  $R_t < 2$  indicate that control could be attained by preventing over half of the secondary transmissions per primary case.

Discuss this quote, in the context of geometric sequences. In particular: why would "preventing over half" control the outbreak? What must they mean by "control?"

- (c) On the next page is the graph from the Nishiura-Chowell paper. How do you think they determined the value of  $R_t$ ? Do you think this outbreak fits the standard geometric model?

**FIGURE 1**

Cumulative and daily epidemic curves of Ebola virus disease (EVD) in Guinea, Liberia, and Sierra Leone, 23 March–26 August 2014



A) Cumulative number of confirmed or probable cases of EVD reported to the World Health Organization [10]. Solid lines are the smoothing spline fits to cumulative curves for each country with a coefficient of variation  $R^2$  at 0.995.

B) Estimated daily incidence curves based on the smoothing spline model. Data from Nigeria and Senegal have been omitted due to the limited number of cases recorded in these countries thus far.

**Question 7.** The Results section of the Nisihura-Chowell paper begins like this this:

### Results

Figure 1 illustrates the process of deriving daily EVD incidence curves by country from cumulative curves of reported cases. Multiple fluctuations are evident from the incidence curve for Guinea (Figure 1). In Liberia, the early transmission phase did not appear to exhibit sustained growth and was probably driven by case importations during first epidemic month. Exponential growth was subsequently seen, reflecting self-sustaining transmission. Similarly, the incidence curve for Sierra Leone also displayed steady growth since early June. Most recent EVD incidence data for Guinea also showed an increasing pattern.

Our weekly maximum likelihood estimates of the  $R_t$  for each affected country and for the global system in West Africa are displayed in Figure 2. Results indicate that the reproduction number for all countries reached levels below unity in April and May, but has appeared to be continuously above one since early June (Figure 2A). This pattern was robust when using two different datasets (including and excluding suspected cases). Estimates of  $R_t$  using total case reports from June to July 2014, a period during which exponential growth of cases has been observed in Sierra Leone and Liberia, ranged from 1.4 to 1.7, respectively. In the hypothetical worst-case scenario that the current situation with an estimated reproduction number  $R$  ranging from 1.4 to 1.7 continues for the remainder of the year, we would expect to observe a total of 77,181 to 277,124 additional cases within 2014

Here are some additional topics for discussion.

- (a) Why is there such a variation in the number of additional expected cases?
- (b) Which parts of the data are fit by a geometric model? Which aren't?
- (c) Epidemiologists have been using these kinds of models for generations. How would you determine whether or not the models are "good?"
- (d) Sometimes the data clearly doesn't fit the geometric curve. Why do people continue to use this model if it doesn't fit all the data?
- (e) In general, the mathematical models used to predict biological events are heavily simplified, and therefore prone to error. Why do we use them?