

Math 225 · 26 Sept 2013 · Discussion Questions

Exercise 1. (a) Calculate: $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} =$

(b) Calculate: $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} =$

(c) The two examples above illustrate that matrix multiplication differs from scalar multiplication in what important way?

(d) Calculate $\begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$. Are they the same?

Exercise 2. (a) Calculate: $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} -2 & -2 \\ 2 & 2 \end{bmatrix} =$

(b) The example above illustrates that matrix multiplication differs from scalar multiplication in what important way?

(c) Calculate: $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(d) Is it true that $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$?

Exercise 3. Determine whether the following sets of matrices are linearly independent.

(a) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$

Exercise 4. Evaluate each of the following. Hint: by cleverly using properties of matrix algebra (for example, $aB + aC = a(B + C)$), you can save yourself some work, but do make it painfully clear how you're doing your calculations.

(a) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 8 & 9 & 8 \\ 9 & 8 & 9 \\ 8 & 9 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} -8 & -9 & -8 \\ -9 & -8 & -9 \\ -8 & -9 & -8 \end{bmatrix}$

(b) $\left(\begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \right) \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

(c) $3.75 \begin{bmatrix} 7 & 4 & 7 \\ 5 & -2 & -7 \\ -2 & -2 & 8 \end{bmatrix} + 3.75 \begin{bmatrix} -7 & -4 & -7 \\ -5 & 2 & 7 \\ 2 & 102 & -8 \end{bmatrix}$

(d) $2.7 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 0.7 \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(f) $\left(\left(\left(\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}^T \right)^T \right)^T \right)^T$

$$(g) \begin{bmatrix} 0 & 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 5 & 9 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 9 & 9 & 2 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 7 & 7 & 1 & 0 & 0 \\ 8 & 5 & 0 & 1 & 0 \\ 0 & 8 & 0 & 0 & 1 \\ 0 & 0 & 9 & 5 & 4 \\ 0 & 0 & 8 & 8 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercise 5. The following table gives the monthly cell-phone usage for the members of three couples.

Name	Local Minutes	Long-Distance Minutes	Texts
Aisha	100	200	100
Ava	300	0	150
Binta	50	50	0
Ben	100	50	0
Chen	100	300	250
Camilia	200	400	200

The major cell phone provider has three different individual rate plans, detailed below.

Name	Local Minutes	Long-Distance Minutes	Texts
Lonesome Traveler Plan	30¢	30¢	free
Happy Homebody Plan	5¢	70¢	2¢
Chatterbox Plan	5¢	30¢	10¢

The major cell phone provider also has three different family rate plans.

Name	Local Minutes	Long-Distance Minutes	Texts
Jet-Setter Family Plan	20¢	20¢	free
Homestead Family Plan	5¢	80¢	1¢
Gregarious Family Plan	5¢	20¢	10¢

Use matrix operations (matrix addition, scalar multiplication, matrix multiplication, transposition) to do the following. Carefully note *which* operations you're using. Make sure your answers make sense!

- Make a table of the monthly **costs** for Aisha, Binta, and Chen under the three individual plans.
- Make a table of the monthly **demands** for the three families (this table will have three rows for the three couples, and three columns for the family plans).
- Make a table of the monthly **costs** for the three families under the family plans.
- Make a table of the monthly **costs** for the three families under the family plans in dollars (not cents).
- (optional!) Make a table of the monthly **costs** for the three families under all *six* plans. Choose the best plan for each family.