Predator Prey

Consider an ecosystem consisting of two species – a predator (like foxes) and its prey (like rabbits). Denote by $y_1(t)$ the size of the predator population at time t and by $y_2(t)$ the size of the prey population at time t. If there are no predators and if food is plentiful, then each member of the prey population causes, on average, a net increase of b_1 per unit time in the size of the population. That is $\frac{dy_1}{dt}(t) = b_1 y_1(t)$. As the size of the prey population grows the amount of food available per member decreases and consequently the net birthrate also decreases, say to $b_1 - f_1 y_1(t)$. Taking this into account, $\frac{dy_1}{dt} = b_1 y_1(t) - f_1 y_1(t)^2$. The corresponding equation for the rate of change of the size of the predator population, in the absence of prey population is $\frac{dy_2}{dt} = -b_2 y_2(t) - f_2 y_2(t)^2$. Note that the net birthrate, $-b_2 - f_2 y_2(t)$, is negative even for very small populations and becomes even more negative for larger populations because of the absence of the prey's primary food. If there are both predators and prey present, the prey net birthrate should decrease by an amount proportional to the average frequency with which each member of prey population encounters a predator. This frequency should increase with the number of predators. There should be a similar increase in the predator net birthrate. Hence

$$\frac{dy_1}{dt} = b_1 y_1 - f_1 y_1^2 - e_1 y_1 y_2$$
$$\frac{dy_2}{dt} = b_2 y_2 - f_2 y_2^2 + e_2 y_1 y_2$$