Important Taylor Series

You should know or be able to quickly rederive the following important Taylor series. (They are also given in Theorem 3.6.6 of the course notes.)

$$
\begin{aligned}
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\
&= 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \cdots && \text{for all } -\infty < x < \infty \\
\sin(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n+1)!} x^{2n+1} \\
&= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots && \text{for all } -\infty < x < \infty \\
\cos(x) &= \sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{2n} \\
&= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots && \text{for all } -\infty < x < \infty \\
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n \\
&= 1 + x + x^2 + x^3 + \cdots && \text{for all } -1 < x < 1 \\
\log(1+x) &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} \\
&= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} - \cdots && \text{for all } -1 < x \le 1 \\
\arctan x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \\
&= x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots && \text{for all } -1 \le x \le 1
\end{aligned}
$$