

Taylor Expansions

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2!}f''(x_0)(x - x_0)^2 + \cdots + \frac{1}{n!}f^{(n)}(x_0)(x - x_0)^n \\ + \frac{1}{(n+1)!}f^{(n+1)}(c)(x - x_0)^{n+1} \quad \text{for some } c \text{ between } x_0 \text{ and } x$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n \\ = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + \cdots$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1} \\ = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + \cdots$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}x^{2n} \\ = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots + \frac{(-1)^n}{(2n)!}x^{2n} + \cdots$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \\ = 1 + x + x^2 + x^3 + \cdots + x^n + \cdots$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n \\ = 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots$$

$$\ln(1-x) = - \sum_{n=1}^{\infty} \frac{1}{n}x^n \\ = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \cdots - \frac{1}{n}x^n - \cdots$$

$$\ln(1+x) = - \sum_{n=1}^{\infty} \frac{(-1)^n}{n}x^n \\ = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots - \frac{(-1)^n}{n}x^n - \cdots$$

$$(1+x)^p = 1 + px + \frac{p(p-1)}{2}x^2 + \frac{p(p-1)(p-2)}{3!}x^3 + \cdots + \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!}x^n + \cdots$$