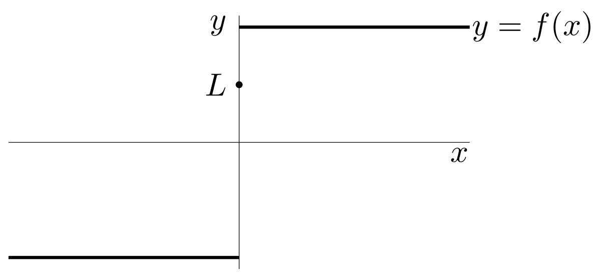
Example of a limit that does not exist

Let

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

I claim that $\lim_{x\to 0} f(x)$ does not exist. To justify this claim, I will show that no matter what number L you pick, $\lim_{x\to 0} f(x)$ does not take the value L. Recall that, in order to have $\lim_{x\to 0} f(x) = L$, $f(x)$ must approach L whenever x approachs 0.



- As an example, let $L = \frac{1}{2}$. Then, for all negative values of x, no matter how small, $f(x)$ takes the value -1, which is nowhere near $L = \frac{1}{2}$. So x can approach zero without $f(x)$ approaching $\frac{1}{2}$ at the same time. So $\lim_{x\to 0} f(x)$ cannot be $\frac{1}{2}$.
- Now consider any $L \geq 0$. Again, for all negative values of x, no matter how small, $f(x)$ takes the value -1, which is still nowhere near the positive number L. So $f(x)$ does not approach L as x tends to zero from the left hand side.
- Finally consider any $L < 0$. Now, for all positive values of x, no matter how small, $f(x)$ takes the value $+1$, which is nowhere near the negative number L. So, as x tends to zero from the right hand side, $f(x)$ does not approach this L either.

We have shown that no matter what L you pick, it is possible for x to approach zero without $f(x)$ approaching L. So, no matter what L you pick, $\lim_{x\to 0} f(x)$ does not take the value L. So $\lim_{x\to 0} f(x)$ does not exist.