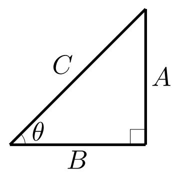
Trig Functions

Definitions

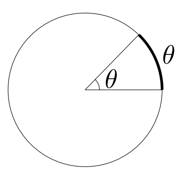


$$\sin\theta = \frac{A}{C} \quad \cos\theta = \frac{B}{C} \quad \tan\theta = \frac{A}{B}$$

$$\csc\theta = \frac{C}{A} \quad \sec\theta = \frac{C}{B} \quad \cot\theta = \frac{B}{A}$$

Radians

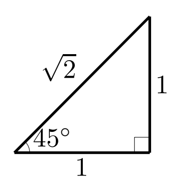
For use in calculus, angles are best measured in units called radians. By definition, an arc of length θ on a circle of radius one subtends an angle of θ radians at the center of the circle. Because the circumference of a circle of radius one is 2π, we have

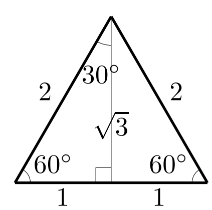


$$2\pi \text{ radians } = 360^\circ \quad \pi \text{ radians } = 180^\circ \quad \tfrac{\pi}{2} \text{ radians } = 90^\circ$$

$$\tfrac{\pi}{3} \text{ radians } = 60^\circ \quad \tfrac{\pi}{4} \text{ radians } = 45^\circ \quad \tfrac{\pi}{6} \text{ radians } = 30^\circ$$

Special Triangles





From the triangles above, we have

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$0^\circ = 0$ rad	0	1	0		1	
$30^\circ = \frac{\pi}{6}$ rad	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
$45^\circ = \frac{\pi}{4}$ rad	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60^\circ = \frac{\pi}{3}$ rad	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
$90^\circ = \frac{\pi}{2}$ rad	1	0		1		0
$180^\circ = \pi$ rad	0	-1	0		-1	

The empty boxes mean that the trig function is undefined (i.e. $\pm\infty$) for that angle.

Trig Identities – Elementary

The following identities are all immediate consequences of the definitions at the top of the previous page

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\cos\theta}{\sin\theta}$$

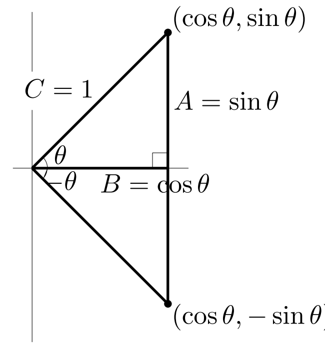
Because 2π radians is $360°$, the angles θ and $\theta + 2\pi$ are really the same, so

$$\sin(\theta + 2\pi) = \sin\theta \qquad \cos(\theta + 2\pi) = \cos\theta$$

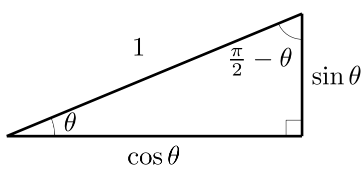
The following trig identities are consequences of the figure to their right.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin(-\theta) = -\sin(\theta) \qquad \cos(-\theta) = \cos(\theta)$$



The following trig identities are consequences of the figure to their left.



$$\sin\left(\tfrac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\tfrac{\pi}{2} - \theta\right) = \sin\theta$$

Trig Identities – Addition Formulae

The following trig identities are derived in the handout entitled "Trig Identities – Cosine law and Addition Formulae"

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\sin(x - y) = \sin x \cos y - \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x - y) = \cos x \cos y + \sin x \sin y$$

Setting $y = x$ gives

$$\sin(2x) = 2\sin x \cos x$$

$$\begin{aligned} \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \qquad \text{since } \sin^2 x = 1 - \cos^2 x \\ &= 1 - 2\sin^2 x \qquad \text{since } \cos^2 x = 1 - \sin^2 x \end{aligned}$$

Solving $\cos(2x) = 2\cos^2 x - 1$ for $\cos^2 x$ and $\cos(2x) = 1 - 2\sin^2 x$ for $\sin^2 x$ gives

$$\cos^2 x = \frac{1+\cos(2x)}{2}$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$