

**Example III.25** (Feldman's notes)

$$\det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 0 & 3 \\ 1 & 3 & 5 & 6 \\ 1 & 3 & 3 & 9 \end{bmatrix} \stackrel{A}{=} \det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 0 & -2 & -1 \\ 0 & 1 & 4 & 4 \\ 0 & 1 & 2 & 7 \end{bmatrix} \begin{array}{l} (1) \\ (2) - 2(1) \\ (3) - (1) \\ (4) - (1) \end{array}$$

$$\stackrel{E}{=} - \det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & -2 & -1 \\ 0 & 1 & 2 & 7 \end{bmatrix} \begin{array}{l} (1) \\ (3) \\ (2) \\ (4) \end{array}$$

$$\stackrel{A}{=} - \det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & -2 & 3 \end{bmatrix} \begin{array}{l} (1) \\ (2) \\ (3) \\ (4) - (2) \end{array}$$

$$\stackrel{A}{=} - \det \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 4 & 4 \\ 0 & 0 & -2 & -1 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{array}{l} (1) \\ (2) \\ (3) \\ (4) - (3) \end{array}$$

$$\stackrel{D}{=} - 1 \times 1 \times (-2) \times 4 = 8$$