Here is a flow chart showing the general algorithm. We use e to stand for "equation number" and v to stand for "variable number".

Set $e = 1$.

Let v be the smallest index such that x_v has nonzero coefficient in an equation (e') with $e' \geq e$.

yes

Does any variable x_v have nonzero coefficient in any equation (e') with $e' \geq e$?

no

Backsolve.

no

yes

Does the variable x_v have nonzero coefficient in equation (e)?

yes

Use $(i) \to (i) + c_i(e)$ to eliminate x_v from every equation (i) with $i > e$.

Increase e by 1. Is (e) the last equation?

no

Interchange equation (e) with a later equation in which x_v has nonzero coefficient.