

MATHEMATICS 200 December 2006 Final Exam Answers

1. (a) $\frac{\partial z}{\partial x} = \frac{4+yz^2}{3z^2-2xyz}$, $\frac{\partial z}{\partial x} = \frac{xz^2}{3z^2-2xyz}$ (b) $\frac{\partial z}{\partial x}(1, 1) = 1$, $\frac{\partial z}{\partial y}(1, 1) = \frac{1}{2}$ (c) ± 0.04
 (d) At A , $\frac{dz}{d\theta} = \frac{2}{3}$. At B , $\frac{dz}{d\theta} = -1$.

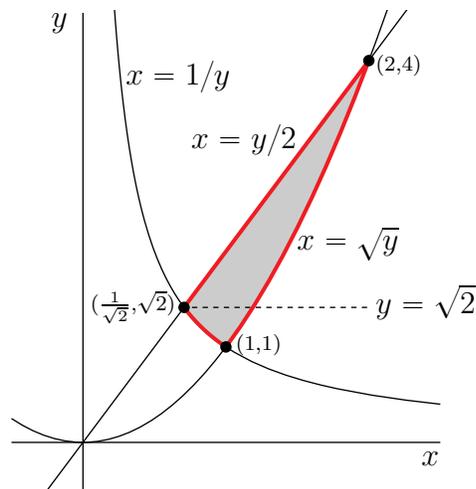
2. (a) The path of steepest ascent is in the direction $-\frac{1}{\sqrt{17}} \langle 1, 4 \rangle$, which is a little west of south. The slope is $|\nabla f(2, 1)| = |\langle -1, -4 \rangle| = \sqrt{17}$.
 (b) So the hiker descends with slope 4.
 (c) $\pm \frac{1}{\sqrt{17}} \langle 4, -1 \rangle$

3. (a)

critical point	type
(0, 0)	saddle point
$(-\frac{2}{3}, \frac{2}{3})$	local max

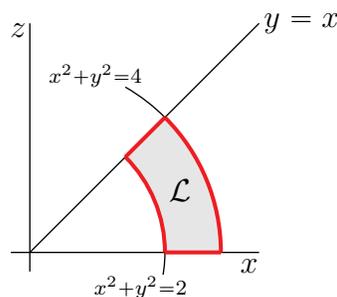
- (b) The maximum and minimum values of f are $\frac{1}{2\sqrt{2}}$ and $-\frac{1}{2\sqrt{2}}$, respectively.

4. (a) The region R is the shaded region in the figure



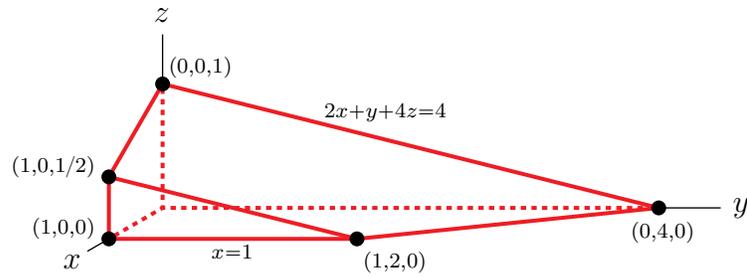
(b) $I = \int_{1/\sqrt{2}}^1 \int_{1/x}^{2x} f(x, y) dy dx + \int_1^2 \int_{x^2}^{2x} f(x, y) dy dx$ (c) $\frac{1}{2}$

5. (a)



(b) $M = \int_0^{\pi/4} d\theta \int_{\sqrt{2}}^2 dr r \rho(r \cos \theta, r \sin \theta)$ (c) $\frac{1}{2}$

6. (a) (i)



(a) (ii)

$$J = \int_{y=0}^{y=2} \int_{x=0}^{x=1} \int_{z=0}^{z=\frac{4-2x-y}{4}} f(x, y, z) dz dx dy + \int_{y=2}^{y=4} \int_{x=0}^{x=\frac{4-y}{4}} \int_{z=0}^{z=\frac{4-2x-y}{4}} f(x, y, z) dz dx dy$$

(b) 2π