

MATHEMATICS 200 December 2008 Final Exam Answers

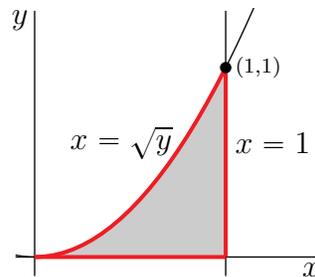
1. (a) $2ax - 2ay + z = -a^2$ (b) $a = \frac{1}{2}$.
2. Method 1 is better.
3. (a) $\frac{\partial u}{\partial x}(x, y) = \frac{e^y}{x}$, $\frac{\partial u}{\partial y}(x, y) = e^y \ln(x) - y^2 e^y - 2ye^y$
 (b) See the solutions.
4. (a) 10 (b) $\pm \frac{1}{5} \langle 3, -4, 0 \rangle$

5. (a)

critical point	$f_{xx}f_{yy} - f_{xy}^2$	f_{xx}	type
$(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$	$(2\sqrt{3} + 2) \times (2) - (-2)^2 > 0$	$2\sqrt{3} + 2 > 0$	local min
$-(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$	$(-2\sqrt{3} + 2) \times (2) - (-2)^2 < 0$		saddle point

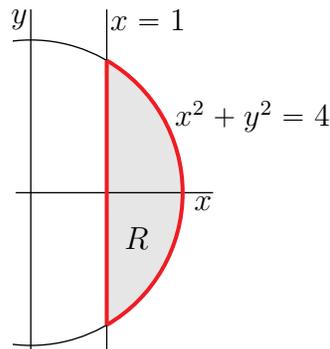
(b) $(1, -2, 1)$ is the closest point. $(-1, 4, -1)$ is the farthest point.

6. (a)



(b) $\frac{1}{\pi}$

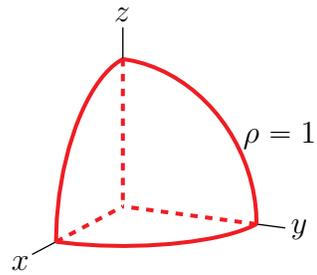
7. (a)



(b) $\text{mass} = \frac{4\pi}{3} - 2 \ln(2 + \sqrt{3})$

(c) $\bar{x} = \frac{2\sqrt{3} - \ln(2 + \sqrt{3})}{\frac{4\pi}{3} - 2 \ln(2 + \sqrt{3})} \approx 1.38$, $\bar{y} = 0$.

8. (a)



(b) $I = \int_0^{\pi/2} d\varphi \int_0^{\pi/2} d\theta \int_0^1 d\rho \rho^4 \sin^2 \varphi \cos \varphi \cos \theta$

(c) $\frac{1}{15}$