MATHEMATICS 200 December 2009 Final Exam Answers

1. (a) $\frac{\partial z}{\partial x} = \frac{y^2z^2}{4z^3-2xy^2z}$, $\frac{\partial z}{\partial y} = \frac{2xyz^2-1}{4z^3-2xy^2z}$

   (b) $\frac{\partial z}{\partial x}(2,-1/2) = \frac{1}{12}$, $\frac{\partial z}{\partial y}(2,-1/2) = -1$

   (c) $f(1.94,-0.4) - 1 \approx -0.105$

   (d) $\frac{x}{12} - y - z = -\frac{1}{3}$

2.

critical point	type
$(0,0)$	local max
$(2,0)$	saddle point

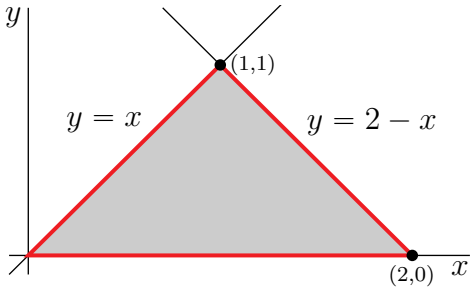
3. (a) The maximum and minimum values of $T(x,y)$ in $x^2+y^2 \le 4$ are 20 (at $(0,0)$) and 4 (at $(0,\pm 2)$), respectively.

   (b) $\frac{1}{\sqrt{17}}\langle -4,-1\rangle$ (c) 18 (d) $(0,2)$

4. $\sqrt{11}$

5. $\frac{1-e^{-1}}{2}$

6. (a)



   (b) $I = \int_0^1 \mathrm{d}x \int_0^x \mathrm{d}y\ \frac{y}{x} + \int_1^2 \mathrm{d}x \int_0^{2-x} \mathrm{d}y\ \frac{y}{x}$ (c) $2\ln 2 - 1$

7. $\bar{x} = \frac{10}{3\pi+8} \approx 0.57$

8. (a) $\int_0^{\pi/2} \mathrm{d}\varphi \int_0^{\pi/2} \mathrm{d}\theta \int_0^1 \mathrm{d}\rho\ \rho^4 \sin^2\varphi \cos\varphi\ \cos\theta$ (b) $\int_0^1 \mathrm{d}z \int_0^{\pi/2} \mathrm{d}\theta \int_0^{\sqrt{1-z^2}} \mathrm{d}r\ r^2\, z\, \cos\theta$

   (c) $\frac{1}{15}$