

# MATHEMATICS 200 December 2009 Final Exam

1. A surface is defined implicitly by  $z^4 - xy^2z^2 + y = 0$ .

(a) Compute  $\frac{\partial z}{\partial x}$ ,  $\frac{\partial z}{\partial y}$  in terms of  $x$ ,  $y$ ,  $z$ .

(b) Evaluate  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  at  $(x, y, z) = (2, -1/2, 1)$ .

(c) If  $x$  decreases from 2 to 1.94, and  $y$  increases from  $-0.5$  to  $-0.4$ , find the approximate change in  $z$  from 1.

(d) Find the equation of the tangent plane to the surface at the point  $(2, -1/2, 1)$ .

2. For the surface

$$z = f(x, y) = x^3 + xy^2 - 3x^2 - 4y^2 + 4$$

Find and classify [as local maxima, local minima, or saddle points] all critical points of  $f(x, y)$ .

3. The temperature  $T(x, y)$  at a point of the  $xy$ -plane is given by

$$T(x, y) = 20 - 4x^2 - y^2$$

(a) Find the maximum and minimum values of  $T(x, y)$  on the disk  $D$  defined by  $x^2 + y^2 \leq 4$ .

(b) Suppose an ant lives on the disk  $D$ . If the ant is initially at point  $(1, 1)$ , in which direction should it move so as to increase its temperature as quickly as possible?

(c) Suppose that the ant moves at a velocity  $\mathbf{v} = \langle -2, -1 \rangle$ . What is its rate of increase of temperature as it passes through  $(1, 1)$ ?

(d) Suppose the ant is constrained to stay on the curve  $y = 2 - x^2$ . Where should the ant go if it wants to be as warm as possible?

4. Use Lagrange multipliers to find the minimum distance from the origin to all points on the intersection of the curves

$$g(x, y, z) = x - z - 4 = 0$$
$$\text{and } h(x, y, z) = x + y + z - 3 = 0$$

5. Find the volume ( $V$ ) of the solid bounded above by the surface

$$z = f(x, y) = e^{-x^2},$$

below by the plane  $z = 0$  and over the triangle in the  $xy$ -plane formed by the lines  $x = 1$ ,  $y = 0$  and  $y = x$ .

6. Consider the integral  $I = \int_0^1 \int_y^{2-y} \frac{y}{x} dx dy$ .

- (a) Sketch the region of integration.
- (b) Interchange the order of integration.
- (c) Evaluate  $I$ .

7. A thin plate of uniform density 1 is bounded by the positive  $x$  and  $y$  axes and the cardioid  $\sqrt{x^2 + y^2} = r = 1 + \sin \theta$ , which is given in polar coordinates. Find the  $x$ -coordinate of its centre of mass.

8. Let

$$I = \iiint_E xz \, dV$$

where  $E$  is the eighth of the sphere  $x^2 + y^2 + z^2 \leq 1$  with  $x, y, z \geq 0$ .

- (a) Express  $I$  as a triple integral in spherical coordinates.
- (b) Express  $I$  as a triple integral in cylindrical coordinates.
- (c) Evaluate  $I$  by any method.