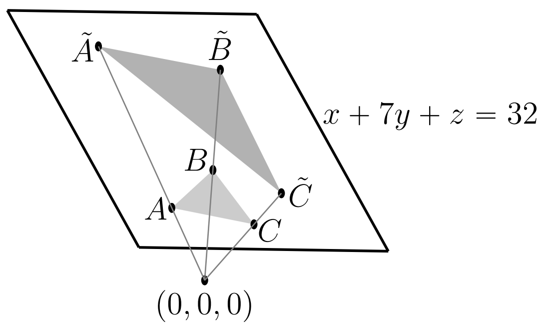
MATHEMATICS 200 December 2016 Final Exam

1. Let $A = (0, 2, 2)$, $B = (2, 2, 2)$, $C = (5, 2, 1)$.

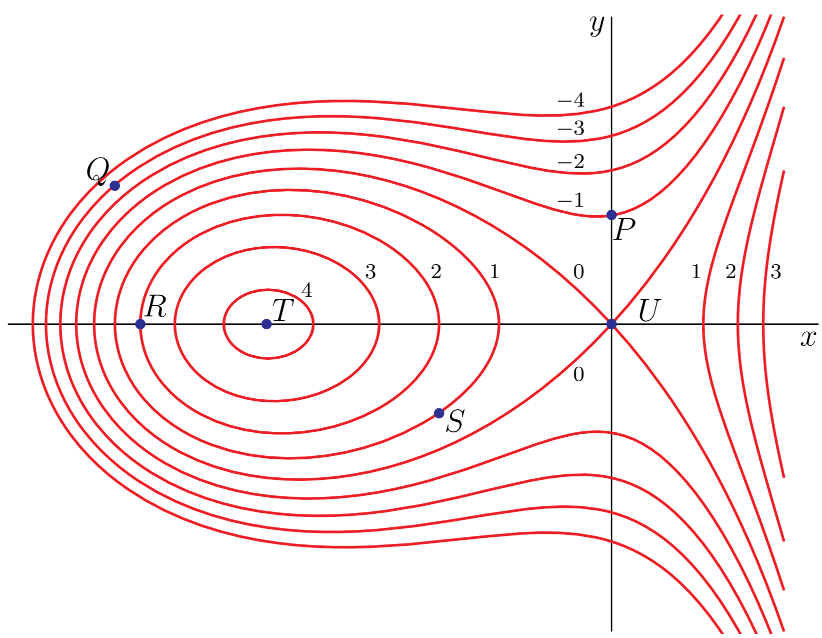
   (a) Find the parametric equations for the line which contains A and is perpendicular to the triangle ABC.

   (b) Find the equation of the set of all points P such that $\overrightarrow{PA}$ is perpendicular to $\overrightarrow{PB}$. This set forms a Plane/Line/Sphere/Cone/Paraboloid/Hyperboloid (circle one) in space.

   (c) A light source at the origin shines on the triangle ABC making a shadow on the plane $x + 7y + z = 32$. (See the diagram.) Find $\tilde{A}$.



2. (a) Some level curves of a function $f(x, y)$ are plotted in the xy–plane below.



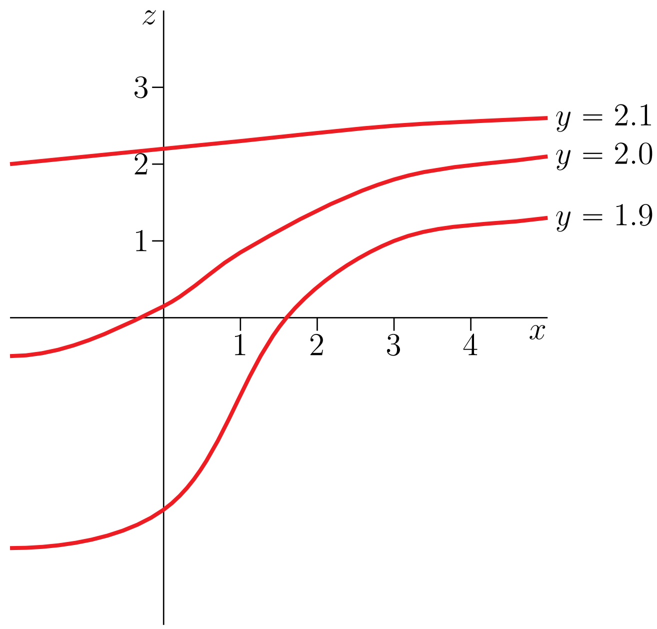
For each of the four statmements below, circle the letters of **all** points in the diagram where the situation applies. For example, if the statement were "These points are on the

y–axis", you would circle both P and U, but none of the other letters. You may assume that a local maximum occurs at point T.

(i)	∇f is zero	P R S T U
(ii)	f has a saddle point	P R S T U
(iii)	the partial derivative f_y is positive	P R S T U
(iv)	the directional derivative of f in the direction $\langle 0, -1\rangle$ is negative	P R S T U

(b) The diagram below shows three "y traces" of a graph $z = F(x, y)$ plotted on xz–axes. (Namely the intersections of the surface $z = F(x, y)$ with the three planes ($y = 1.9$, $y = 2$, $y = 2.1$). For each statement below, circle the correct word.

(i)	the first order partial derivative $F_x(1, 2)$ is	positive/negative/zero (circle one)
(ii)	F has a critical point at $(2, 2)$	true/false (circle one)
(iii)	the second order partial derivative $F_{xy}(1, 2)$ is	positive/negative/zero (circle one)



3. Consider the functions $F(x, y, z) = z^3 + xy^2 + xz$ and $G(x, y, z) = 3x - y + 4z$. You are standing at the point $P(0, 1, 2)$.

(a) You jump from P to $Q(0.1\,,\,0.9\,,\,1.8)$. Use the linear approximation to determine approximately the amount by which F changes.

(b) You jump from P in the direction along which G increases most rapidly. Will F increase or decrease?

(c) You jump from P in a direction $\langle a\,,\,b\,,\,c\rangle$ along which the rates of change of F and G are both zero. Give an example of such a direction (need not be a unit vector).

4. Suppose that $f(x,y)$ is twice differentiable (with $f_{xy} = f_{yx}$), and $x = r\cos\theta$ and $y = r\sin\theta$.

   (a) Evaluate f_θ, f_r and $f_{r\theta}$ in terms of r, θ and partial derivatives of f with respect to x and y.

   (b) Let $g(x,y)$ be another function satisfying $g_x = f_y$ and $g_y = -f_x$. Express f_r and f_θ in terms of r, θ and g_r, g_θ.

5. The temperature in the plane is given by $T(x,y) = e^y\left(x^2 + y^2\right)$.

   (a) (i) Give the system of equations that must be solved in order to find the warmest and coolest point on the circle $x^2 + y^2 = 100$ by the method of Lagrange multipliers.

   (ii) Find the warmest and coolest points on the circle by solving that system.

   (b) (i) Give the system of equations that must be solved in order to find the critical points of $T(x,y)$.

   (ii) Find the critical points by solving that system.

   (c) Find the coolest point on the solid disc $x^2 + y^2 \leq 100$.

6. Let $I = \int_0^1 \int_{x^2}^1 x^3 \ \sin(y^3) \ \mathrm{d}y \ \mathrm{d}x$.

   (a) Sketch the region of integration in the xy–plane. Label your sketch sufficiently well that one could use it to determine the limits of double integration.

   (b) Evaluate I.

7. Let S be the region on the first octant (so $x, y, z \geq 0$) which lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $(z-1)^2 + x^2 + y^2 = 1$. Let V be its volume.

   (a) Express V as a triple integral in cylindrical coordinates.

   (b) Express V as an triple integral in spherical coordinates.

   (c) Calculate V using either of the integrals above.

8. Let E be the region bounded by the planes $y = 0$, $y = 2$, $y + z = 3$ and the surface $z = x^2$. Consider the intergal

$$I = \iiint_E f(x,y,z) \ \mathrm{d}V$$

   Fill in the blanks below. In each part below, you may need only one integral to express your answer. In that case, leave the other blank.

   (a) $I = \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} f(x,y,z) \ \mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y + \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} f(x,y,z) \ \mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y$

(b) $I = \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} f(x,y,z)\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z + \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} f(x,y,z)\ \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

(c) $I = \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} f(x,y,z)\ \mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z + \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} \int_{\_\_}^{\_\_} f(x,y,z)\ \mathrm{d}y\,\mathrm{d}x\,\mathrm{d}z$