Multivariables Calculus Warnings

These notes highlight number of common, but serious, multivariable calculus errors.

Warning 1. A line in three dimensions has infinitely many normal vectors.

Discussion. For example, the line

$$(x, y, z) = (1, 1, 0) + t(1, 2, -2)$$

has direction vector $(1, 2, -2)$. Any vector perpendicular to $(1, 2, -2)$ is perpendicular to the line. The vector (n_1, n_2, n_3) is perpendicular to $(1, 2, -2)$ if and only if

$$0 = (1, 2, -2) \cdot (n_1, n_2, n_3) = n_1 + 2n_2 - 2n_3$$

There is whole plane of (n_1, n_2, n_3)'s obeying this condition, of which $(2, -1, 0)$, $(0, 1, 1)$ and $(2, 0, 1)$ are only three examples.

Warning 2. The cross product has two properties that are very different from the corresponding properties for the multiplication of real numbers.

$$\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$$
$$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$$

for most $\vec{a}$, $\vec{b}$ and $\vec{c}$. For example

$$\hat{\imath} \times (\hat{\imath} \times \hat{\jmath}) = \hat{\imath} \times \hat{k} = -\hat{k} \times \hat{\imath} = -\hat{\jmath}$$
$$(\hat{\imath} \times \hat{\imath}) \times \hat{\jmath} = \vec{0} \times \hat{\jmath} = \vec{0}$$

