Taylor Expansions

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\begin{aligned}
f(x) &= f(x_0) + f'(x_0)\,(x-x_0) + \tfrac{1}{2!}f''(x_0)\,(x-x_0)^2 + \cdots + \tfrac{1}{n!}f^{(n)}(x_0)\,(x-x_0)^n \\
&\qquad + \tfrac{1}{(n+1)!}f^{(n+1)}(c)\,(x-x_0)^{n+1} \qquad \text{for some } c \text{ between } x_0 \text{ and } x \\
e^x &= \sum_{n=0}^{\infty} \tfrac{1}{n!}x^n \\
&= 1 + x + \tfrac{1}{2}x^2 + \tfrac{1}{3!}x^3 + \cdots + \tfrac{1}{n!}x^n + \cdots \\
\sin x &= \sum_{n=0}^{\infty} \tfrac{(-1)^n}{(2n+1)!}x^{2n+1} \\
&= x - \tfrac{1}{3!}x^3 + \tfrac{1}{5!}x^5 - \cdots + \tfrac{(-1)^n}{(2n+1)!}x^{2n+1} + \cdots \\
\cos x &= \sum_{n=0}^{\infty} \tfrac{(-1)^n}{(2n)!}x^{2n} \\
&= 1 - \tfrac{1}{2!}x^2 + \tfrac{1}{4!}x^4 - \cdots + \tfrac{(-1)^n}{(2n)!}x^{2n} + \cdots \\
\tfrac{1}{1-x} &= \sum_{n=0}^{\infty} x^n \\
&= 1 + x + x^2 + x^3 + \cdots + x^n + \cdots \\
\tfrac{1}{1+x} &= \sum_{n=0}^{\infty} (-1)^n x^n \\
&= 1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots \\
\ln(1-x) &= -\sum_{n=1}^{\infty} \tfrac{1}{n}x^n \\
&= -x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3 - \cdots - \tfrac{1}{n}x^n - \cdots \\
\ln(1+x) &= -\sum_{n=1}^{\infty} \tfrac{(-1)^n}{n}x^n \\
&= x - \tfrac{1}{2}x^2 + \tfrac{1}{3}x^3 - \cdots - \tfrac{(-1)^n}{n}x^n - \cdots \\
(1+x)^p &= 1 + px + \tfrac{p(p-1)}{2}x^2 + \tfrac{p(p-1)(p-2)}{3!}x^3 + \cdots + \tfrac{p(p-1)(p-2)\cdots(p-n+1)}{n!}x^n + \cdots
\end{aligned}
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