Vector Identities

Gradient

T 1. $\nabla(f + g) = \nabla f + \nabla g$

T 2. $\nabla(cf) = c\nabla f$, for any **constant** c

E 3. $\nabla(fg) = f\nabla g + g\nabla f$

E 4. $\nabla(f/g) = (g\nabla f - f\nabla g)/g^2$ at points $\mathbf{x}$ where $g(\mathbf{x}) \neq 0$.

Pcc 5. $\nabla(\mathbf{F} \cdot \mathbf{G}) = \mathbf{F} \times (\nabla \times \mathbf{G}) - (\nabla \times \mathbf{F}) \times \mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} + (\mathbf{F} \cdot \nabla)\mathbf{G}$

Divergence

T 6. $\nabla \cdot (\mathbf{F} + \mathbf{G}) = \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G}$

T 7. $\nabla \cdot (c\mathbf{F}) = c\nabla \cdot \mathbf{F}$, for any **constant** c

E , Pa 8. $\nabla \cdot (f\mathbf{F}) = f\nabla \cdot \mathbf{F} + \mathbf{F} \cdot \nabla f$

Pb 9. $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$

Curl

T 10. $\nabla \times (\mathbf{F} + \mathbf{G}) = \nabla \times \mathbf{F} + \nabla \times \mathbf{G}$

T 11. $\nabla \times (c\mathbf{F}) = c\nabla \times \mathbf{F}$, for any **constant** c

E 12. $\nabla \times (f\mathbf{F}) = f\nabla \times \mathbf{F} + \nabla f \times \mathbf{F}$

Pc 13. $\nabla \times (\mathbf{F} \times \mathbf{G}) = \mathbf{F}(\nabla \cdot \mathbf{G}) - (\nabla \cdot \mathbf{F})\mathbf{G} + (\mathbf{G} \cdot \nabla)\mathbf{F} - (\mathbf{F} \cdot \nabla)\mathbf{G}$

Laplacian

T 14. $\nabla^2(f + g) = \nabla^2 f + \nabla^2 g$

T 15. $\nabla^2(cf) = c\nabla^2 f$, for any **constant** c

E 16. $\nabla^2(fg) = f\nabla^2 g + 2\nabla f \cdot \nabla g + g\nabla^2 f$

Degree Two

Pe 17. $\nabla \cdot (\nabla \times \mathbf{F}) = 0$

Pf 18. $\nabla \times (\nabla f) = 0$

Pd 19. $\nabla \cdot \big[f(\nabla g \times \nabla h)\big] = \nabla f \cdot (\nabla g \times \nabla h)$

Paa 20. $\nabla \cdot (f\nabla g - g\nabla f) = f\nabla^2 g - g\nabla^2 f$

Pg 21. $\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$

Code:

T: Trivial (proof $\leq$ 1 line).

E: Easy (proof $\leq$ 3 lines).

Paa: Easy by #8.

Pa: Problem Set VII, problem 6(a).

Pb: Problem Set VII, problem 6(b).

Pc: Problem Set VII, problem 6(c).

Pcc: Similar to Problem Set VII, problem 6(c).

Pd: Problem Set VII, problem 6(d).

Pe: Problem Set VII, problem 6(e). See also Problem Set VII, problem 8.

Pf: Problem Set VII, problem 6(f).

Pg: Problem Set VII, problem 6(g).