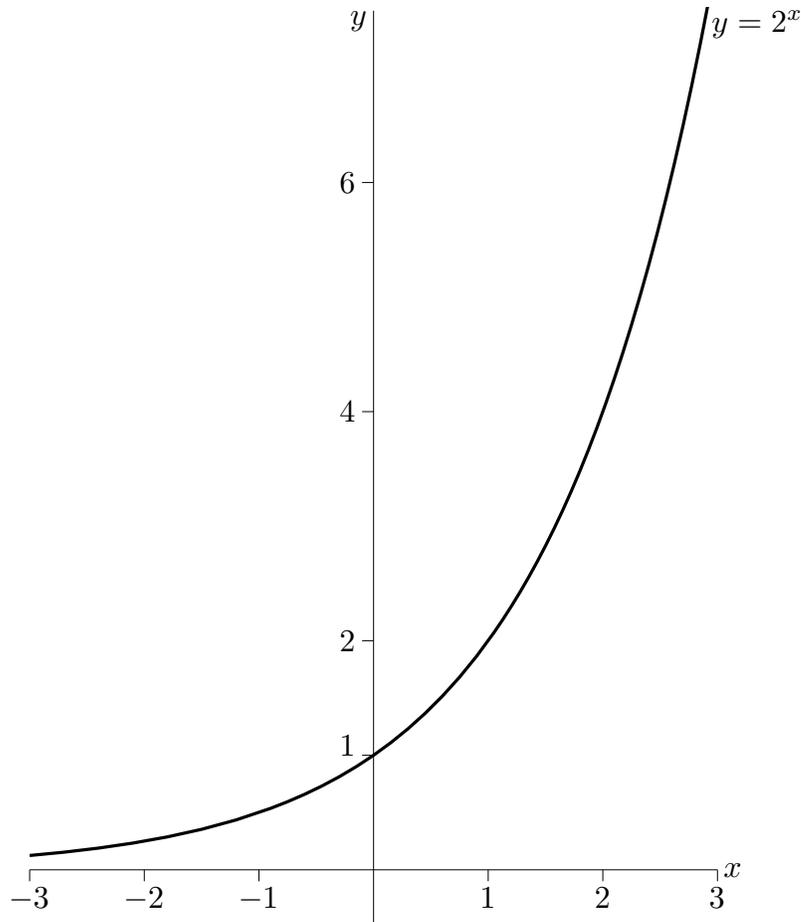# Properties of Exponentials

In the following, x and y are arbitrary real numbers, a and b are arbitrary constants that are strictly bigger than zero and e is 2.7182818284, to ten decimal places.

1) $e^0 = 1, \quad a^0 = 1$

2) $e^{x+y} = e^x e^y, \quad a^{x+y} = a^x a^y$

3) $e^{-x} = \frac{1}{e^x}, \quad a^{-x} = \frac{1}{a^x}$

4) $\left(e^x\right)^y = e^{xy}, \quad \left(a^x\right)^y = a^{xy}$

5) $\frac{d}{dx}e^x = e^x, \quad \frac{d}{dx}e^{g(x)} = g'(x)e^{g(x)}, \quad \frac{d}{dx}a^x = (\ln a)\, a^x$

6) $\int e^x \, dx = e^x + C, \quad \int e^{ax} \, dx = \frac{1}{a}e^{ax} + C$ if $a \neq 0$

7) $\lim\limits_{x \to \infty} e^x = \infty, \quad \lim\limits_{x \to -\infty} e^x = 0$

   $\lim\limits_{x \to \infty} a^x = \infty, \quad \lim\limits_{x \to -\infty} a^x = 0$ if $a > 1$

   $\lim\limits_{x \to \infty} a^x = 0, \quad \lim\limits_{x \to -\infty} a^x = \infty$ if $0 < a < 1$

8) The graph of 2^x is given below. The graph of a^x, for any $a > 1$, is similar.

Properties of Logarithms

In the following, x and y are arbitrary real numbers that are strictly bigger than 0, a is an arbitrary constant that is strictly bigger than one and e is 2.7182818284, to ten decimal places.

1) $e^{\ln x} = x$, $a^{\log_a x} = x$, $\log_e x = \ln x$, $\log_a x = \frac{\ln x}{\ln a}$

2) $\log_a (a^x) = x$, $\ln (e^x) = x$

    $\ln 1 = 0$, $\log_a 1 = 0$

    $\ln e = 1$, $\log_a a = 1$

3) $\ln(xy) = \ln x + \ln y$, $\log_a(xy) = \log_a x + \log_a y$

4) $\ln \left(\frac{x}{y}\right) = \ln x - \ln y$, $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

    $\ln \left(\frac{1}{y}\right) = -\ln y$, $\log_a \left(\frac{1}{y}\right) = -\log_a y$,

5) $\ln(x^y) = y \ln x$, $\log_a(x^y) = y \log_a x$

6) $\frac{d}{dx} \ln x = \frac{1}{x}$, $\frac{d}{dx} \ln(g(x)) = \frac{g'(x)}{g(x)}$, $\frac{d}{dx} \log_a x = \frac{1}{x \ln a}$

7) $\int \frac{1}{x} dx = \ln |x| + C$, $\int \ln x \ dx = x \ln x - x + C$

8) $\lim_{x \to \infty} \ln x = \infty$, $\lim_{x \to 0} \ln x = -\infty$

    $\lim_{x \to \infty} \log_a x = \infty$, $\lim_{x \to 0} \log_a x = -\infty$

9) The graph of $\ln x$ is given below. The graph of $\log_a x$, for any $a > 1$, is similar.