

# Formulae for $dS$ and $\hat{n} dS$

## Main Formulae

For the **parametrized surface**  $\vec{r}(u, v)$

$$\hat{n} dS = \pm \frac{\partial \vec{r}}{\partial u}(u, v) \times \frac{\partial \vec{r}}{\partial v}(u, v) \, dudv$$

$$dS = \left| \frac{\partial \vec{r}}{\partial u}(u, v) \times \frac{\partial \vec{r}}{\partial v}(u, v) \right| \, dudv$$

For the **graph**  $z = f(x, y)$

$$\hat{n} dS = \pm \left[ -f_x(x, y)\hat{\mathbf{i}} - f_y(x, y)\hat{\mathbf{j}} + \hat{\mathbf{k}} \right] \, dxdy$$

$$dS = \sqrt{1 + f_x(x, y)^2 + f_y(x, y)^2} \, dxdy$$

For the **level surface**  $F(x, y, z) = 0$

$$\hat{n} dS = \pm \frac{\vec{\nabla} F(x, y, z(x, y))}{\vec{\nabla} F(x, y, z(x, y)) \cdot \hat{\mathbf{k}}} \, dxdy$$

$$dS = \left| \frac{\vec{\nabla} F(x, y, z(x, y))}{\vec{\nabla} F(x, y, z(x, y)) \cdot \hat{\mathbf{k}}} \right| \, dxdy$$

# Variations

For the surface  $x = g(y, z)$

$$\hat{n} dS = \pm [\hat{\mathbf{i}} - g_y(y, z)\hat{\mathbf{j}} - g_z(y, z)\hat{\mathbf{k}}] dydz$$

$$dS = \sqrt{1 + g_y(y, z)^2 + g_z(y, z)^2} dydz$$

For the surface  $y = h(x, z)$

$$\hat{n} dS = \pm [-h_x(x, z)\hat{\mathbf{i}} + \hat{\mathbf{j}} - h_z(x, z)\hat{\mathbf{k}}] dxdz$$

$$dS = \sqrt{1 + h_x(x, z)^2 + h_z(x, z)^2} dxdz$$

For the **level surface**  $F(x, y, z) = 0$

$$\hat{n} dS = \pm \frac{\vec{\nabla} F(x, y, z(x, y))}{\vec{\nabla} F(x, y, z(x, y)) \cdot \hat{\mathbf{k}}} dx dy$$

$$= \pm \frac{\vec{\nabla} F(x, y(x, z), z)}{\vec{\nabla} F(x, y(x, z), z) \cdot \hat{\mathbf{j}}} dx dz$$

$$= \pm \frac{\vec{\nabla} F(x(y, z), y, z)}{\vec{\nabla} F(x(y, z), y, z) \cdot \hat{\mathbf{i}}} dy dz$$

$$dS = \left| \frac{\vec{\nabla} F(x, y, z(x, y))}{\vec{\nabla} F(x, y, z(x, y)) \cdot \hat{\mathbf{k}}} \right| dx dy$$

$$= \left| \frac{\vec{\nabla} F(x, y(x, z), z)}{\vec{\nabla} F(x, y(x, z), z) \cdot \hat{\mathbf{j}}} \right| dx dz$$

$$= \left| \frac{\vec{\nabla} F(x(y, z), y, z)}{\vec{\nabla} F(x(y, z), y, z) \cdot \hat{\mathbf{i}}} \right| dy dz$$