

# Example of Newtons Method

$$\begin{aligned}f(x, y) &= 1 + (x - 2)^4 + (x - 2)^2 y^2 + (y + 1)^2 \\ \nabla f(x, y) &= \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} 4(x - 2)^3 + 2(x - 2)y^2 \\ 2(x - 2)^2 y + 2(y + 1) \end{bmatrix} \\ H(x, y) &= \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} = \begin{bmatrix} 12(x - 2)^2 + 2y^2 & 4(x - 2)y \\ 4(x - 2)y & 2(x - 2)^2 + 2 \end{bmatrix} \\ \vec{x}_{k+1} &= \vec{x}_k - H(\vec{x}_k)^{-1} \nabla f(\vec{x}_k)\end{aligned}$$

## Initialization

$$\vec{x}_0 = \langle 1, 1 \rangle \quad \text{MATLAB: } X = [1; 1];$$

## First Step

Math	MATLAB
$\nabla f(1, 1) = \begin{bmatrix} -6 \\ 6 \end{bmatrix}$	<code>G = gradf(X);</code>
$H(1, 1) = \begin{bmatrix} 14 & -4 \\ -4 & 4 \end{bmatrix}$	<code>H = hessf(X);</code>
$\vec{x}_1 - \vec{x}_0 = -H(1, 1)^{-1} \nabla f(1, 1) = \langle 0, -\frac{3}{2} \rangle$	<code>dX = -H \setminus G ;</code>
$\vec{x}_1 = \langle 1, -0.5 \rangle$	<code>X = X + dX ;</code>

## First Eight Steps

$k$	$x_k$	$y_k$
0	1.0000	1.0000
1	1.0000	-0.5000
2	1.3913	-0.6957
3	1.7459	-0.9488
4	1.9863	-1.0482
5	1.9987	-1.0002
6	2.0000	-1.0000
7	2.0000	-1.0000
8	2.0000	-1.0000