Discrete Probability Distributions

Uniform Distribution

Experiment obeys: all outcomes equally probable
Random variable: outcome
Probability distribution: if k is the number of possible outcomes,

$$p(x) = \begin{cases} \frac{1}{k} & \text{if } x \text{ is a possible outcome} \\ 0 & \text{otherwise} \end{cases}$$

Example: tossing a fair die ($k = 6$)

Bernoulli Distribution

Experiment obeys: (1) a single trial with two possible outcomes (success and failure)
(2) $P\Big(\{\text{trial is successful}\}\Big) = p$
Random variable: number of successful trials (zero or one)
Probability distribution: $p(x) = p^x(1-p)^{n-x}$
Mean and variance: $\mu = p$, $\sigma^2 = p(1-p)$
Example: tossing a fair coin once

Binomial Distribution

Experiment obeys: (1) n repeated trials
(2) each trial has two possible outcomes (success and failure)
(3) $P\Big(\{i^{\text{th}} \text{ trial is successful}\}\Big) = p$ for all i
(4) the trials are independent
Random variable: number of successful trials
Probability distribution: $b(x; n, p) = \binom{n}{x} p^x(1-p)^{n-x}$
Mean and variance: $\mu = np$, $\sigma^2 = np(1-p)$
Example: tossing a fair coin n times
Approximations: (1) $b(x; n, p) \approx p(x; \lambda = pn)$ if $p \ll 1,\ x \ll n$ (Poisson approximation)
(2) $b(x; n, p) \approx n(x; \mu = pn, \sigma = \sqrt{np(1-p)}\,)$ if $np \gg 1,\ n(1-p) \gg 1$
(Normal approximation)

Geometric Distribution

Experiment obeys: (1) indeterminate number of repeated trials
(2) each trial has two possible outcomes (success and failure)
(3) $P\Big(\{i^{\text{th}} \text{ trial is successful}\}\Big) = p$ for all i
(4) the trials are independent
Random variable: trial number of first successful trial
Probability distribution: $p(x) = p(1-p)^{x-1}$
Mean and variance: $\mu = \frac{1}{p}$, $\sigma^2 = \frac{1-p}{p^2}$
Example: repeated attempts to start an engine, or playing a lottery until you win

Negative Binomial Distribution

Experiment obeys: (1) indeterminate number of repeated trials
(2) each trial has two possible outcomes (success and failure)
(3) $P\Big(\{i^{\text{th}} \text{ trial is successful}\}\Big) = p$ for all i
(4) the trials are independent
(5) keep going until r^{th} success
Random variable: trial number on which r^{th} success occurs
Probability distribution: $b^*(x; r, p) = \binom{x-1}{r-1} p^r(1-p)^{x-r}$
Mean and variance: $\mu = \frac{r}{p}$, $\sigma^2 = \frac{r(1-p)}{p^2}$
Example: fabricating r nondefective computer chips

Poisson Distribution

Experiment obeys: count the number of occurrences of some event in a specified time interval or in a specified region of space where:

(1) the events occur at a point in time or space

(2) the number of events occurring in one region is independent of the number occurring in any disjoint region

(3) the probability of more than one event occurring at the same point is negligible

(4) the probability of n events in region #1 is the same as the probability of n events in region #2, when the regions have the same size

Random variable: number of events occurring in the given time interval or region of space

Probability distribution: $p(x;\lambda) = \frac{e^{-\lambda}\lambda^x}{x!}$ where λ is the average number of events in the given region

Mean and variance: $\mu = \lambda$, $\sigma^2 = \lambda$

Example: telephone calls arriving at a switchboard in a specified one hour period

Hypergeometric Distribution

Experiment obeys: (1) a random sample of size n is selected from N items

(2) there are k items of one type (called successes) and $N-k$ items of another type (called failures)

Random variable: number of successes selected

Probability distribution: $h(x;N,n,k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$

Mean and variance: $\mu = n\frac{k}{N}$, $\sigma^2 = \frac{N-n}{N-1}n\frac{k}{N}\left(1-\frac{k}{N}\right)$

Example: selecting a random sample of 5 spark plugs from a batch of 40 of which 3 are defective