

Computation of the Poisson Distribution

$$\begin{aligned}
 P(Y = y) &= \lim_{n \rightarrow \infty} \binom{n}{y} \left(\frac{\lambda}{n}\right)^y \left(1 - \frac{\lambda}{n}\right)^{n-y} \\
 &= \lim_{n \rightarrow \infty} \frac{n!}{y!(n-y)!} \left(\frac{\lambda}{n}\right)^y \left(1 - \frac{\lambda}{n}\right)^{n-y} \\
 &= \lim_{n \rightarrow \infty} \frac{\lambda^y}{y!} \frac{n(n-1)\cdots(n-y+1)}{n \cdot n \cdots n} \left(1 - \frac{\lambda}{n}\right)^{n-y} \\
 &= \lim_{n \rightarrow \infty} \frac{\lambda^y}{y!} \frac{n(n-1)\cdots(n-y+1)}{n \cdot n \cdots n} \frac{\left(1 - \frac{\lambda}{n}\right)^{-y}}{\left(1 - \frac{\lambda}{n}\right)^n} \\
 &= \lim_{n \rightarrow \infty} \frac{\lambda^y}{y!} \frac{1}{1 - \frac{\lambda}{n}} \frac{1 - \frac{1}{n}}{1 - \frac{\lambda}{n}} \cdots \frac{1 - \frac{y-1}{n}}{1 - \frac{\lambda}{n}} \left(1 - \frac{\lambda}{n}\right)^n \\
 &= \frac{\lambda^y}{y!} \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n \\
 &= e^{-\lambda} \frac{\lambda^y}{y!}
 \end{aligned}$$