Computation of the Poisson Distribution

$$P(Y = y) \ = \ \lim_{n \to \infty} \binom{n}{y} \left(\frac{\lambda}{n}\right)^y \left(1 - \frac{\lambda}{n}\right)^{n-y}$$

$$= \ \lim_{n \to \infty} \frac{n!}{y!(n-y)!} \left(\frac{\lambda}{n}\right)^y \left(1 - \frac{\lambda}{n}\right)^{n-y}$$

$$= \ \lim_{n \to \infty} \frac{\lambda^y}{y!} \frac{n}{n} \frac{(n-1)}{n} \cdots \frac{(n-y+1)}{n} \left(1 - \frac{\lambda}{n}\right)^{n-y}$$

$$= \ \lim_{n \to \infty} \frac{\lambda^y}{y!} \frac{n}{n} \frac{(n-1)}{n} \cdots \frac{(n-y+1)}{n} \left(1 - \frac{\lambda}{n}\right)^{-y} \left(1 - \frac{\lambda}{n}\right)^n$$

$$= \ \lim_{n \to \infty} \frac{\lambda^y}{y!} \frac{1}{1 - \frac{\lambda}{n}} \frac{1 - \frac{1}{n}}{1 - \frac{\lambda}{n}} \cdots \frac{1 - \frac{y-1}{n}}{1 - \frac{\lambda}{n}} \left(1 - \frac{\lambda}{n}\right)^n$$

$$= \ \frac{\lambda^y}{y!} \lim_{n \to \infty} \left(1 - \frac{\lambda}{n}\right)^n$$

$$= \ e^{-\lambda} \frac{\lambda^y}{y!}$$