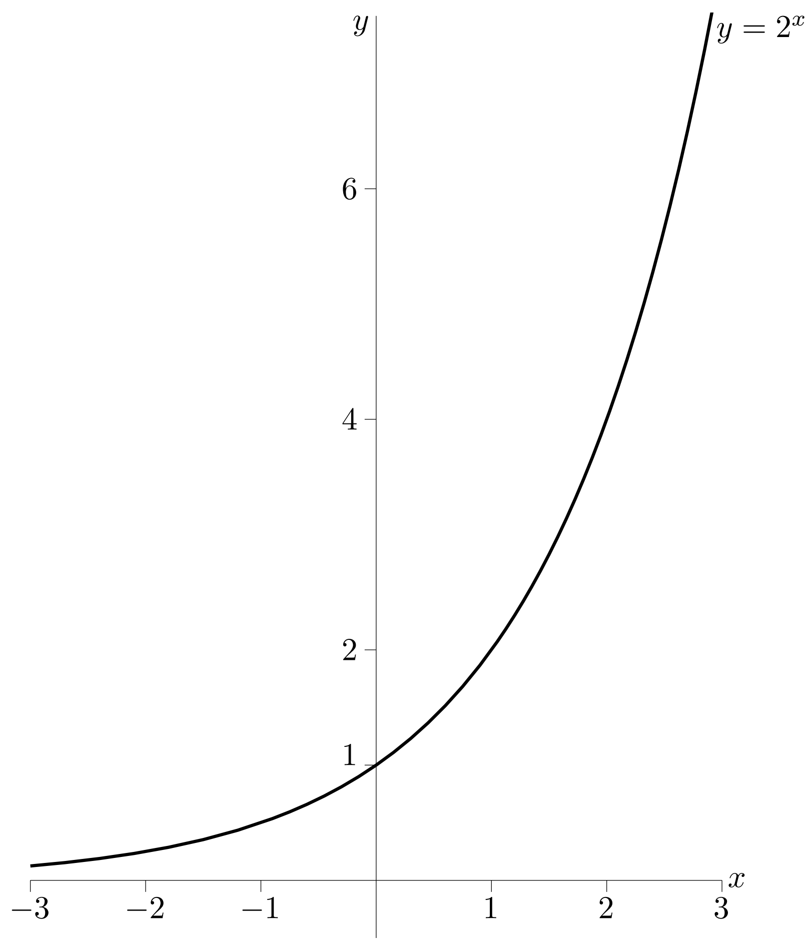
# Properties of Exponentials

In the following, $x$ and $y$ are arbitrary real numbers, $a$ and $b$ are arbitrary constants that are strictly bigger than zero and $e$ is 2.7182818284, to ten decimal places.

1) $e^0 = 1,\ \ a^0 = 1$
2) $e^{x+y} = e^x e^y,\ \ a^{x+y} = a^x a^y$
3) $e^{-x} = \frac{1}{e^x},\ \ a^{-x} = \frac{1}{a^x}$
4) $\big(e^x\big)^y = e^{xy},\ \big(a^x\big)^y = a^{xy}$
5) $\frac{d}{dx}e^x = e^x,\ \ \frac{d}{dx}e^{g(x)} = g'(x)e^{g(x)},\ \ \frac{d}{dx}a^x = (\ln a)\ a^x$
6) $\int e^x\ dx = e^x + C,\ \ \int e^{ax}\ dx = \frac{1}{a}e^{ax} + C$ if $a \neq 0$
7) $\lim\limits_{x\to\infty} e^x = \infty,\ \lim\limits_{x\to-\infty} e^x = 0$
   $\lim\limits_{x\to\infty} a^x = \infty,\ \lim\limits_{x\to-\infty} a^x = 0$ if $a > 1$
   $\lim\limits_{x\to\infty} a^x = 0,\ \lim\limits_{x\to-\infty} a^x = \infty$ if $0 < a < 1$
8) The graph of $2^x$ is given below. The graph of $a^x$, for any $a > 1$, is similar.



# Properties of Logarithms

In the following, $x$ and $y$ are arbitrary real numbers that are strictly bigger than 0, $a$ is an arbitrary constant that is strictly bigger than one and $e$ is 2.7182818284, to ten decimal places.

1) $e^{\ln x} = x, \quad a^{\log_a x} = x, \quad \log_e x = \ln x, \quad \log_a x = \frac{\ln x}{\ln a}$
2) $\log_a \big(a^x\big) = x, \ \ln \big(e^x\big) = x$

   $\ln 1 = 0, \quad \log_a 1 = 0$

   $\ln e = 1, \quad \log_a a = 1$
3) $\ln(xy) = \ln x + \ln y, \quad \log_a(xy) = \log_a x + \log_a y$
4) $\ln\left(\frac{x}{y}\right) = \ln x - \ln y, \quad \log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

   $\ln\left(\frac{1}{y}\right) = -\ln y, \quad \log_a\left(\frac{1}{y}\right) = -\log_a y,$
5) $\ln(x^y) = y \ln x, \quad \log_a(x^y) = y \log_a x$
6) $\frac{d}{dx}\ln x = \frac{1}{x}, \ \frac{d}{dx}\ln(g(x)) = \frac{g'(x)}{g(x)}, \ \frac{d}{dx}\log_a x = \frac{1}{x \ln a}$
7) $\int \frac{1}{x}dx = \ln|x| + C, \ \int \ln x \ dx = x\ln x - x + C$
8) $\lim\limits_{x\to\infty} \ln x = \infty, \ \lim\limits_{x\to 0} \ln x = -\infty$

   $\lim\limits_{x\to\infty} \log_a x = \infty, \ \lim\limits_{x\to 0} \log_a x = -\infty$
9) The graph of $\ln x$ is given below. The graph of $\log_a x$, for any $a > 1$, is similar.