Important Vector Identities

Linearity

1. $\boldsymbol{\nabla}(f+g) = \boldsymbol{\nabla}f + \boldsymbol{\nabla}g$
2. $\boldsymbol{\nabla}(cf) = c\,\boldsymbol{\nabla}f$, for any **constant** c

3. $\boldsymbol{\nabla}\cdot(\mathbf{F}+\mathbf{G}) = \boldsymbol{\nabla}\cdot\mathbf{F} + \boldsymbol{\nabla}\cdot\mathbf{G}$
4. $\boldsymbol{\nabla}\cdot(c\mathbf{F}) = c\,\boldsymbol{\nabla}\cdot\mathbf{F}$, for any **constant** c

5. $\boldsymbol{\nabla}\times(\mathbf{F}+\mathbf{G}) = \boldsymbol{\nabla}\times\mathbf{F} + \boldsymbol{\nabla}\times\mathbf{G}$
6. $\boldsymbol{\nabla}\times(c\mathbf{F}) = c\,\boldsymbol{\nabla}\times\mathbf{F}$, for any **constant** c

7. $\boldsymbol{\nabla}^2(f+g) = \boldsymbol{\nabla}^2 f + \boldsymbol{\nabla}^2 g$
8. $\boldsymbol{\nabla}^2(cf) = c\,\boldsymbol{\nabla}^2 f$, for any **constant** c

Product Rule

9. $\boldsymbol{\nabla}(fg) = f\boldsymbol{\nabla}g + g\boldsymbol{\nabla}f$
10. $\boldsymbol{\nabla}\cdot(f\mathbf{F}) = f\boldsymbol{\nabla}\cdot\mathbf{F} + \mathbf{F}\cdot\boldsymbol{\nabla}f$
11. $\boldsymbol{\nabla}\times(f\mathbf{F}) = f\boldsymbol{\nabla}\times\mathbf{F} + \boldsymbol{\nabla}f\times\mathbf{F}$

Screening Tests

12. $\boldsymbol{\nabla}\times(\boldsymbol{\nabla}f) = 0$
13. $\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla}\times\mathbf{F}) = 0$