Urysohn's Lemma

We constructed open sets V_r, $r \in \mathbb{Q} \cap [0,1]$, obeying

$$K \subset V_1 \subset \overline{V_1} \subset \cdots \subset V_s \subset \overline{V_s} \subset \cdots \subset V_r \subset \overline{V_r} \subset \cdots \subset V_0 \subset \overline{V_0} \subset V \subset X$$

for all $0 < r < s < 1$, $r, s \in \mathbb{Q} \cap [0,1]$. Then we defined

$$f(x) = \begin{cases} \sup\left\{ \; r \;\middle|\; r \in \mathbb{Q} \cap [0,1],\ x \in V_r \; \right\} & \text{if } x \in V_0 \\ 0 & \text{if } x \notin V_0 \end{cases}$$