Integration Formulae

In the following

- $f,\ g$ are C^1 functions.
- $\varphi,\ \psi$ are C^2 functions.
- ω is a C^1 1–form.
- D is a region in M with compact closure and piecewise differentiable boundary.

(1) $$d\omega = 0 \quad \Rightarrow \quad \int_{\delta D} \omega = 0$$

(2) $$\int_{\delta D} f\omega = \int_D df \wedge \omega + \int_D f\, d\omega$$

(3) $$f\omega \text{ has compact support contained in } D \quad \Rightarrow \quad \int_D df \wedge \omega + \int_D f\, d\omega = 0$$

(4) $$\big(df, *\omega\big)_D = \int_D f\, d\bar\omega - \int_{\delta D} f\bar\omega$$

(5) $$\big(d\varphi, d\psi\big)_D = -\int_D \varphi\Delta\bar\psi + \int_{\delta D} \varphi * d\bar\psi$$

(6) $$\int_D \big(\varphi\Delta\psi - \psi\Delta\varphi\big) = \int_{\delta D} \big(\varphi * d\psi - \psi * d\varphi\big)$$

(7) $$\big(df, *d\psi\big)_D = -\int_{\delta D} f\, d\bar\psi$$

(8) $$d\omega = 0 \quad \Rightarrow \quad \int_{\delta D} f\bar\omega = \int_D df \wedge \bar\omega$$

(9) $$\int_{\delta D} f\, d\bar g = -\int_{\delta D} \bar g\, df$$

(10) $$d\omega = d(f\omega) = 0 \ \Rightarrow \ \int_D df \wedge \bar\omega = 2\int_{\delta D} (\mathrm{Re}\, f)\, \bar\omega = 2i \int_{\delta D} (\mathrm{Im}\, f)\, \bar\omega$$