Simple ODE Solvers - Summary

These notes provide a summary of three fixed step–size methods for generating, numerically, approximate solutions to the initial value problem

$$y'(t) = f\big(t, y(t)\big)$$
$$y(t_0) = y_0$$

We use $\phi(t)$ to denote the exact solution and $A(t,h)$ to denote the approximation to $\phi(t)$ generated when step size h is used.

Euler's Method

$$y_{n+1} = y_n + f\big(t_n, y_n\big)h$$
$$A(t,h) = \phi(t) + K(t)h + O(h^2)$$

The Improved Euler's Method

$$y_{n+1} = y_n + \tfrac{1}{2}\Big[f\big(t_n, y_n\big) + f\Big(t_{n+1}, y_n + f(t_n, y_n)h\Big)\Big]h$$
$$A(t,h) = \phi(t) + K(t)h^2 + O(h^3)$$

The Runge-Kutta Method

$$k_{n,1} = f(t_n, y_n)$$
$$k_{n,2} = f(t_n + \tfrac{1}{2}h, y_n + \tfrac{h}{2}k_{n,1})$$
$$k_{n,3} = f(t_n + \tfrac{1}{2}h, y_n + \tfrac{h}{2}k_{n,2})$$
$$k_{n,4} = f(t_n + h, y_n + hk_{n,3})$$
$$y_{n+1} = y_n + \tfrac{h}{6}\left[k_{n,1} + 2k_{n,2} + 2k_{n,3} + k_{n,4}\right]$$
$$A(t,h) = \phi(t) + K(t)h^4 + O(h^5)$$