ASSIGNMENT 3

---

1. One important component of problem solving is to extract general insights from specific mistakes.

   Find one problem that you solved incorrectly on a previous assignment. In one paragraph, describe the specific mistake you made. In a second paragraph, describe a general modification you can make to your problem solving approach to avoid making similar mistakes in the future.

   (If you achieved a perfect score on all of your previous assignments, describe a general part of your problem solving approach that helps you do so well!)

The *rectifier* function

$$r(x) = \left\{ \begin{array}{ll} 0 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{array} \right\}$$

is used in artificial neural networks to model the firing of neurons. However, $r(x)$ is not differentiable at 0. Differentiability can improve the stability and performance of neural networks. Two common differentiable approximations to $r(x)$ are the *softplus* function

$$p(x) = \log(1 + e^x)$$

and the *swish* function

$$s(x) = \frac{x}{1 + e^{-x}}.$$

In this assignment, you may use without proof the facts that $p(x) > r(x)$ and $s(x) \leq r(x)$ for all x, and $p(x)$, $r(x)$ and $s(x)$ are continuous.

2. (a) Explain why $p(x)$ approximates $r(x)$ well for large (positive and negative) values of x.

   (b) Explain why $s(x)$ approximates $r(x)$ well for large (positive and negative) values of x.

3. Where is $p(x)$ the worst approximation to $r(x)$? In other words, where is the vertical distance between the two functions maximized?

4. (a) On the interval $(-\infty, 0)$, where is $s(x)$ the worst approximation to $r(x)$? You may not be able to determine an exact x-value, but find the integer $a < 0$ such that $s(x)$ is the worst approximation to $r(x)$ somewhere in the interval $[a, a+1]$.

   You are encouraged to use a program like Desmos to find a; but to get full marks, you must also justify your choice of a rigorously — for example, by using calculus.

   (b) On the interval $(0, \infty)$, where is $s(x)$ the worst approximation to $r(x)$? You may not be able to determine an exact x-value, but find the integer $b \geq 0$ such that $s(x)$ is the worst approximation to $r(x)$ somewhere in the interval $[b, b+1]$.

   You are encouraged to use a program like Desmos to find b; but to get full marks, you must also justify your choice of b rigorously — for example, by using calculus.