Math 612: Single Cell Analysis **2019W Term 1**

Lecture 7: September 26, 2019

Lecturer: Geoffrey Schiebinger *Scribe: John Edgar*

7.1 Developmental Stochastic Processes

Previously:

- A population of cells is represented by a distribution P
- P can represent an organism or tissue or ecosystem at the time of sampling

This lecture:

Def: A stochastic process is an indexed set of random variables $x_t : t \in T$ where $x_t \in X$ is a random process and T is an index set.

Example 1: sequence of independent coin flips

$$F_i = \begin{cases} 1 & \text{w/ prob } 1/2 \\ 0 & \text{w/ prob } 1/2 \end{cases}$$

Example 2: random walks $x_0 = 0$

$$x_{t+1} = \begin{cases} x_t + 1 & \text{w/ prob } 1/2 \\ x_t - 1 & \text{w/ prob } 1/2 \end{cases}$$

Def: the lineage tree of a developing population is a binary tree with a leaf for every live cell present. Shows the history of cell division.

Def: differentation is the process by which two sets of cells (subpopulations) diverge to create two different cell types (fates). This can be represented by Waddington's Landscape.

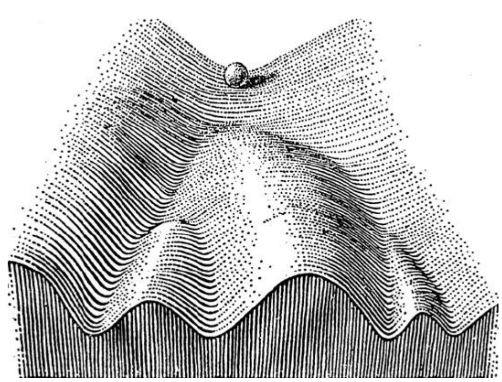

Figure 7.1: Waddinton's Landscape

Def: a stochastic process is Markov or memoryless if the future evolution depends only on the present state, and not the history leading up to the current state.

Example: yearly rainfall in Vancouver $R_{2001}, R_{2002}, R_{2003}$

7.1.1 Continuous Time Process

Poisson process $w_1, w_2, ...w_n$ where w_i is an exponential random variable:

$Pr(w_i > t) = e^{-\lambda t} \approx 1 - \lambda t$ for small t and where λ is the rate of the process.

Def: $S_n = \sum_{i=1}^{n} w_i$ is a Poisson process $N(t) = n$ if $S_n \leq t$ and $S_{n+1} > t$.

Example: a birth-death process is a stochastic process modelling the number of individuals $N(t)$ in a population.

$$N(t) = \begin{cases} N(t) + 1 = N(t + dt) & \text{w/ prob} = \beta dt \\ N(t) & \text{w/ prob} \approx 1 - \beta dt - \delta dt \\ N(t) - 1 = N(t + dt) & \text{w/ prob} = \delta dt \end{cases}$$

Starting with $N(0)$ individuals at time 0, the expected population size is $\mathbb{E}\, N(t) = e^{(\beta - \delta)t}$.

Next class: continuous time, continuous state Markov processes