University of British Columbia
Quarter Putnam Contest—November 7, 2017

Your solutions should be clear, concise, and complete. *"Educated guesses" will earn little credit.*
Write your solutions as if this were the actual Putnam examination.

QP1. Sonequa and Marcel repeatedly flip a coin and keep track of the sequence of heads and tails that result. (The coin is a fair coin, which lands heads 50% of the time and tails 50% of the time, and all flips are independent of one another.) They play until either three flips in a row give heads-tails-tails in that order, in which case Sonequa wins, or three flips in a row give heads-heads-tails in that order, in which case Marcel wins. Determine, with justification, whether Sonequa has a better chance of winning, Marcel has a better chance of winning, or their chances are equal.

QP2. Evaluate

$$\frac{1}{2^1+1} + \frac{2}{2^2+1} + \frac{4}{2^4+1} + \frac{8}{2^8+1} + \frac{16}{2^{16}+1} + \cdots .$$

QP3. Let $\mathcal{S}$ be a fixed set of points in the plane. Given a point P in the plane but not itself in $\mathcal{S}$, a *buddy* is a point $Q \in \mathcal{S}$ such that the distance from Q to P is strictly less than the distance from Q to any other point in $\mathcal{S}$. (Note that the definition concerns itself with the distances between Q and other points in $\mathcal{S}$, not between P and other points in $\mathcal{S}$. For example, if S is the set of integer points on the x-axis and $P = (0,1)$, then P has no buddies.)

What is the maximum possible number of buddies a point P in the plane can have?