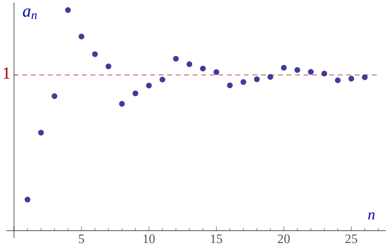
Wednesday, February 27

Clicker Questions

Clicker Question 1

Graph of a sequence

Based on the terms of the sequence you can see, does $\{a_n\}$ converge to 1 or not?



A. no, because there's no formula for the values
B. no, because some values are above 1 while other values are below 1
C. no, because some values are farther away from 1 than previous values
D. yes, because the values will get as close to 1 as we like if we go far enough
E. yes, because each value is closer to 1 than the previous value

Clicker Question 2

Functions and sequences

If a function $f(x)$ is defined for all positive real numbers, we can consider the sequence $\{f(n)\} = \{f(1), f(2), f(3), \ldots\}$. What can we say about the relationship between the limit of the function $\lim_{x\to\infty} f(x)$, and the limit of the sequence $\lim_{n\to\infty} f(n)$?

A. If $\lim_{n\to\infty} f(n)$ converges, then $\lim_{x\to\infty} f(x)$ converges to the same value.

B. $\lim_{x\to\infty} f(x)$ converges to a value exactly when $\lim_{n\to\infty} f(n)$ converges to the same value.

C. There is no reliable relationship between $\lim_{x\to\infty} f(x)$ and $\lim_{n\to\infty} f(n)$.

D. $\lim_{x\to\infty} f(x)$ diverges exactly when $\lim_{n\to\infty} f(n)$ diverges.

E. If $\lim_{x\to\infty} f(x)$ converges, then $\lim_{n\to\infty} f(n)$ converges to the same value.

Clicker Question 3

Applying the Squeeze Theorem

Calculate $\lim_{n\to\infty} \dfrac{(-1)^n + 2n + 3\cos 4n}{n}$.

A. 0
B. 1
C. 2
D. 3
E. 4

Two bounding sequences

Since $(-1)^n$ is either -1 or 1, and $3\cos 4n$ is always between -3 and 3, the limit must lie between

$$\lim_{n\to\infty} \frac{2n-4}{n} \quad \text{and} \quad \lim_{n\to\infty} \frac{2n+4}{n},$$

both of which equal 2.

Clicker Question 4

Will this problem send you to the hospital?

Evaluate $\lim_{n\to\infty} \frac{\ln n}{n^{1/9}}$.

A. converges to 9
B. converges to 1
C. converges to 0
D. diverges
E. converges to $1/9$

Using l'Hospital's Rule

It suffices to calculate $\lim_{x\to\infty} \frac{\ln x}{x^{1/9}}$, which is an $\frac{\infty}{\infty}$ indeterminate form. Its limit is therefore equal to

$$\lim_{x\to\infty} \frac{(\ln x)'}{(x^{1/9})'} = \lim_{x\to\infty} \frac{1/x}{x^{-8/9}/9} = \lim_{x\to\infty} \frac{9}{x^{1/9}} = 0.$$