Clicker Questions

An improper integral (infinite interval)

Determine whether

$$\int_5^\infty \frac{1}{x}\,dx$$

is convergent or divergent; if it is convergent, find its value.

A. $\ln 5$

B. e^5

C. $1/5$

D. divergent

E. none of the above

$$\int_5^\infty \frac{1}{x}\,dx = \lim_{t\to\infty} \int_5^t \frac{1}{x}\,dx$$
$$= \lim_{t\to\infty} \left(\ln t - \ln 5 \right)$$
$$= \infty$$

Clicker Question 2

Another improper integral (infinite interval)

Determine whether

$$\int_{-\infty}^{3} 2^x \, dx$$

is convergent or divergent; if it is convergent, find its value.

A. $8 \ln 2$

B. $8/\ln 2$

C. $-8 \ln 2$

D. divergent

E. none of the above

$$\int_{-\infty}^{3} 2^x \, dx = \lim_{t \to -\infty} \int_{t}^{3} 2^x \, dx$$

$$= \lim_{t \to -\infty} \left(\frac{2^x}{\ln 2} \Big]_{t}^{3} \right)$$

$$= \lim_{t \to -\infty} \left(\frac{2^3}{\ln 2} - \frac{2^t}{\ln 2} \right)$$

$$= \frac{8}{\ln 2}$$

Clicker Question 3

Yet another improper integral (asymptote at an endpoint)

Determine whether

$$\int_0^{\pi/2} \sec^2 x \, dx$$

is convergent or divergent; if it is convergent, find its value.

A. $\sqrt{3}$

B. $\pi/2$

C. 1

D. divergent

E. none of the above

$$\int_0^{\pi/2} \sec^2 x \, dx = \lim_{t \to \frac{\pi}{2}-} \int_0^t \sec^2 x \, dx$$

$$= \lim_{t \to \frac{\pi}{2}-} \left(\tan t - \tan 0 \right)$$

$$= \infty$$