Monday, March 11

Clicker Questions

Clicker Question 1

Practicing the Limit Comparison Test

Determine the convergence or divergence of these two series:

$$\text{I. } \sum_{n=7}^{\infty} \frac{1}{2^n - 100} \qquad \text{II. } \sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{4n^5 + 6}}$$

A. I. diverges but II. converges

B. I. converges but II. diverges

C. both I. and II. diverge

D. both I. and II. converge

Series to compare to

I. $\lim_{n\to\infty} \frac{2^{-n}}{1/(2^n - 100)} = \lim_{n\to\infty} \left(1 - 100 \cdot 2^{-n}\right) = 1$, and the geometric series $\sum_{n=7}^{\infty} 2^{-n}$ converges.

II. $\lim_{n\to\infty} \frac{n^{-5/3}}{1/\sqrt[3]{4n^5 + 6}} = \lim_{n\to\infty} \sqrt[3]{4 + 6n^{-5}} = \sqrt[3]{4}$, and the p-series $\sum_{n=1}^{\infty} n^{-5/3}$ converges.

Clicker Question 2

A family of limits

Consider the function $f(x) = x^k C^{-x}$, where k and C are constants. Which of the following conditions will ensure that $\lim_{x\to\infty} x^k C^{-x} = 0$?

A. $C > k$

B. $x = 0$

C. $C > 1$

D. $k < 0$

E. none of the above

Why C?

"Exponentials always grow/shrink faster than polynomials." One way to justify this: use l'Hospital's Rule on x^k/C^x many times (at least k times in a row).

Why not A or D?

If $k = -1$ and $C = \frac{1}{2}$, we have $\lim_{x\to\infty} x^{-1}(\frac{1}{2})^{-x} = \lim_{x\to\infty} 2^x/x = \infty$.