

Monday, March 18

# Clicker Questions

# Clicker Question 1

## A series with a parameter

Using the Ratio Test, determine values of  $C$  for which the series

$$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{(-3)^n} C^n$$

converges and diverges.

## Bigger or smaller than 1?

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} \\ &= \lim_{n \rightarrow \infty} \frac{\tan^{-1}(n+1) \cdot |C|^{n+1}/3^{n+1}}{\tan^{-1} n \cdot |C|^n/3^n} \\ &= \lim_{n \rightarrow \infty} \left( \frac{\tan^{-1}(n+1)}{\tan^{-1} n} \cdot \frac{|C|}{3} \right) \\ &= \frac{\pi/2}{\pi/2} \frac{|C|}{3} = \frac{|C|}{3}. \end{aligned}$$

- A. converges for  $-\frac{1}{3} < C < \frac{1}{3}$ ; diverges for  $C > \frac{1}{3}$  and  $C < -\frac{1}{3}$
- B. converges for  $0 < C < \frac{1}{3}$ ; diverges for  $C > \frac{1}{3}$  and  $C < 0$
- C. converges for  $-3 < C < 0$ ; diverges for  $C > 0$  and  $C < -3$
- D. converges for  $-3 < C < 3$ ; diverges for  $C > 3$  and  $C < -3$
- E. none of the above

## Clicker Question 2

### An off-centre power series

On which of the following intervals does  $\sum_{n=0}^{\infty} 2n^3(x-5)^n$  converge?

- A.  $4 < x < 6$
- B.  $-6 < x < -4$
- C.  $-6 < x < 6$
- D.  $-1 < x < 1$
- E. none of the above

### Still the Ratio Test

$$\begin{aligned}\lim_{n \rightarrow \infty} \frac{|a_{n+1}|}{|a_n|} &= \lim_{n \rightarrow \infty} \frac{2(n+1)^3|x-5|^{n+1}}{2n^3|x-5|^n} \\ &= \lim_{n \rightarrow \infty} \frac{(n+1)^3}{n^3}|x-5| \\ &= |x-5|.\end{aligned}$$

And  $|x-5| < 1$  precisely when  $4 < x < 6$ .