Monday, March 25

Clicker Questions

Clicker Question 1

Factorials are more intense than exponentials

If you apply the Ratio Test to the series $\sum_{n=0}^{\infty} x^n/n!$, you will find that it converges for every real number x (in other words, its radius of convergence is ∞). Why can we conclude from this that $\displaystyle\lim_{n\to\infty} \frac{x^n}{n!} = 0$ for every real number x?

A. The Squeeze Theorem
B. The Limit Comparison Test
C. l'Hospital's Rule
D. The Integral Test
E. The Test for Divergence

Two statements of the same fact

Given a sequence $\{a_n\}$, the Test for Divergence states that if $\lim_{n\to\infty} a_n \neq 0$ (or if the limit doesn't exist at all), then the series $\sum_{n=0}^{\infty} a_n$ diverges.

In other words, if the series $\sum_{n=0}^{\infty} a_n$ actually converges, then it must be the case that $\lim_{n\to\infty} a_n = 0$.

Clicker Question 2

Finding a Maclaurin series

What is the Maclaurin series for the function $f(x) = e^x$?

A. $\displaystyle\sum_{n=0}^{\infty} \frac{e^x}{n!}$

B. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

C. $\displaystyle\sum_{n=0}^{\infty} x^n$

D. $\displaystyle\sum_{n=0}^{\infty} e^x x^n$

E. $\displaystyle\sum_{n=0}^{\infty} \frac{e^x x^n}{n!}$

The calculation

The Maclaurin series for a function $f(x)$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

In this case, $f^{(n)}(x) = e^x$ for every real number x, and so $f^{(n)}(0) = e^0 = 1$ always.

Clicker Question 3

Finding a Taylor series

The Taylor series for the function $f(x) = 1/(x-5)^3$ centred at $a = 7$ has the form

$$c_0 + c_1(x-7) + c_2(x-7)^2 + c_3(x-7)^3 + c_4(x-7)^4 + \cdots.$$

The five numbers below are c_0, c_1, c_2, c_3, c_4 in some order. Which one is c_3?

A. $\frac{1}{8}$

B. $\frac{3}{16}$

C. $-\frac{3}{16}$

D. $-\frac{5}{32}$

E. $\frac{15}{128}$

The calculation

The Taylor series at a for a function $f(x)$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

so $c_3 = f^{(3)}(7)/3! = f'''(7)/6$. Since

$$f'''(x) = (-3)(-4)(-5)/(x-5)^6,$$

we get $f'''(7) = (-60)/2^6 = -15/16$.