Wednesday, March 27

Clicker Questions

Clicker Question 1

Exploiting a power series representation

We have just derived the power series representation

$$e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$$

for $|x| \le 2$. What is the exact value of the convergent sum that we have seen a couple of times now, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \approx -0.632$?

A. $-\frac{1}{e}$

B. $\frac{1}{e} - 1$

C. $\frac{1}{e}$

D. $1 - e$

E. none of the above

Making sure the indices match

$$e^{-x} - 1 = -1 + \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} x^n.$$

Now plug in $x = 1$ on both sides.

Clicker Question 2

Recognizing power series

Which of the following series converges to $1/2$?

A. $\sum_{n=0}^{\infty} (-1)^n \frac{(\pi/6)^{2n+1}}{(2n+1)!} = \sin \frac{\pi}{6} = \frac{1}{2}$

B. $\sum_{n=0}^{\infty} (-1)^n \frac{(\pi/3)^{2n}}{(2n)!} = \cos \frac{\pi}{3} = \frac{1}{2}$

C. $\sum_{n=0}^{\infty} \frac{(-\ln 2)^n}{n!} = e^{-\ln 2} = \frac{1}{2}$

D. $\sum_{n=1}^{\infty} -\frac{(1-\sqrt{e})^n}{n} = \ln\left(1-(1-\sqrt{e})\right) = \frac{1}{2}$ (note: $|1-\sqrt{e}| < 1$)

E. $\sum_{n=1}^{\infty} \frac{4^{n-3/2}}{5^n} = \frac{4^{-1/2}/5^1}{1-4/5} = \frac{1}{2}$ (note: $|\frac{4}{5}| < 1$)