Wednesday, March 6

Clicker Questions

Clicker Question 1

Staring a series at a different index

Let $\{a_n\}$ be a sequence. Suppose you know that the series $\sum_{n=1}^{\infty} a_n = S$ converges to some value S. What can you say about the series $\sum_{n=4}^{\infty} a_n$?

A. it converges only if a_1, a_2, a_3 are positive

B. it converges only if $a_1 > a_2 > a_3 > a_4$

C. it always converges, to $S - (a_1 + a_2 + a_3)$

D. it always diverges

E. there's not enough information to tell

Two related sums

The partial sums of

$$\sum_{n=4}^{\infty} a_n = a_4 + a_5 + \cdots$$

are all $(a_1 + a_2 + a_3)$ less than the partial sums of

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + a_5 + \cdots .$$

Clicker Question 2

Practicing the Integral Test

Determine the convergence or divergence of these two series:

$$\text{I. } \sum_{n=1}^{\infty} n^2 e^{-n^3} \qquad \text{II. } \frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \frac{1}{14} + \frac{1}{17} + \cdots$$

A. both I. and II. diverge

B. I. converges but II. diverges

C. I. diverges but II. converges

D. both I. and II. converge

The relevant integrals

I. $$\int_1^{\infty} x^2 e^{-x^3}\,dx = \lim_{t\to\infty} \int_1^t x^2 e^{-x^3}\,dx$$
$$= \lim_{t\to\infty} \left(-\tfrac{1}{3}e^{-x^3}\right)\Big]_1^t = 0 + \tfrac{1}{3}e^{-1}.$$

II. equals $\sum_{n=1}^{\infty} 1/(3n+2)$:

$$\int_1^{\infty} \frac{1}{3x+2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{3x+2}\,dx$$
$$= \lim_{t\to\infty} \left(\tfrac{1}{3}\ln|3x+2|\right)\Big]_1^t = \infty.$$

Clicker Question 3

The "p-series"

For which real numbers p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

A. converges for $p < 0$, but diverges for $p \geq 0$

B. diverges for every p

C. converges for $p < 1$, but diverges for $p \geq 1$

D. converges for $p > 0$, but diverges for $p \leq 0$

E. converges for $p > 1$, but diverges for $p \leq 1$

Using the Integral Test

Compare to

$$\int_1^{\infty} \frac{1}{x^p}\,dx = \lim_{t\to\infty} \int_1^t x^{-p}\,dx$$

$$= \lim_{t\to\infty} \left(\frac{t^{1-p}}{1-p} - \frac{1^{1-p}}{1-p} \right)$$

$$= \begin{cases} 0 - \frac{1}{1-p}, & \text{if } 1-p < 0, \\ \infty, & \text{if } 1-p > 0. \end{cases}$$

($p = 1$ is handled separately.)