Wednesday, April 1

Clicker Questions

Clicker Question 1

Finding a Maclaurin series

What is the Maclaurin series for the function $f(x) = e^x$?

A. $\sum_{n=0}^{\infty} x^n$

B. $\sum_{n=0}^{\infty} \frac{e^x x^n}{n!}$

C. $\sum_{n=0}^{\infty} e^x x^n$

D. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

E. $\sum_{n=0}^{\infty} \frac{e^x}{n!}$

The calculation

The Maclaurin series for a function $f(x)$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n.$$

In this case, $f^{(n)}(x) = e^x$ for every real number x, and so $f^{(n)}(0) = e^0 = 1$ always.

Clicker Question 2

Finding a Taylor series

The Taylor series for the function $f(x) = 1/(x-5)^3$ centred at $a = 7$ has the form

$$c_0 + c_1(x-7) + c_2(x-7)^2 + c_3(x-7)^3 + c_4(x-7)^4 + \cdots.$$

The five numbers below are c_0, c_1, c_2, c_3, c_4 in some order. Which one is c_3?

A. $-\frac{5}{32}$

B. $\frac{1}{8}$

C. $\frac{3}{16}$

D. $-\frac{3}{16}$

E. $\frac{15}{128}$

The calculation

The Taylor series at a for a function $f(x)$ is

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n,$$

so $c_3 = f^{(3)}(7)/3! = f'''(7)/6$. Since

$$f'''(x) = (-3)(-4)(-5)/(x-5)^6,$$

we get $f'''(7) = (-60)/2^6 = -15/16$.