Monday, February 23

Clicker Questions

Clicker Question 1

An improper integral (infinite interval)

Determine whether

$$\int_5^\infty \frac{1}{x}\,dx$$

is convergent or divergent; if it is convergent, find its value.

A. divergent
B. $\ln 5$
C. $1/5$
D. e^5
E. $1/25$

$$\begin{aligned}\int_5^\infty \frac{1}{x}\,dx &= \lim_{t\to\infty} \int_5^t \frac{1}{x}\,dx \\ &= \lim_{t\to\infty} \left(\ln t - \ln 5\right) \\ &= \infty\end{aligned}$$

Clicker Question 2

Another improper integral (infinite interval)

Determine whether

$$\int_{-\infty}^{3} 2^x \, dx$$

is convergent or divergent; if it is convergent, find its value.

A. $8 \ln 2$

B. divergent

C. $-8 \ln 2$

D. $-8/\ln 2$

E. $8/\ln 2$

$$\begin{aligned}
\int_{-\infty}^{3} 2^x \, dx &= \lim_{t \to -\infty} \int_{t}^{3} 2^x \, dx \\
&= \lim_{t \to -\infty} \left(\left. \frac{2^x}{\ln 2} \right]_t^3 \right) \\
&= \lim_{t \to -\infty} \left(\frac{2^3}{\ln 2} - \frac{2^t}{\ln 2} \right) \\
&= \frac{8}{\ln 2}
\end{aligned}$$

Clicker Question 3

Yet another improper integral (asymptote at an endpoint)

Determine whether

$$\int_0^{\pi/2} \sec^2 x\,dx$$

is convergent or divergent; if it is convergent, find its value.

A. 1

B. $\pi/2$

C. divergent

D. $\sqrt{3}$

E. none of the above

$$\begin{aligned}\int_0^{\pi/2} \sec^2 x\,dx &= \lim_{t \to \frac{\pi}{2}-} \int_0^t \sec^2 x\,dx \\ &= \lim_{t \to \frac{\pi}{2}-} \left(\tan t - \tan 0\right) \\ &= \infty\end{aligned}$$