Wednesday, March 11

Clicker Questions

Clicker Question 1

Practicing the Integral Test

Determine the convergence or divergence of these two series:

$$\text{I. } \sum_{n=1}^{\infty} n^2 e^{-n^3} \qquad \text{II. } \frac{1}{5} + \frac{1}{8} + \frac{1}{11} + \frac{1}{14} + \frac{1}{17} + \cdots$$

A. both I. and II. diverge

B. I. converges but II. diverges

C. I. diverges but II. converges

D. both I. and II. converge

The relevant integrals (check: both integrands are decreasing)

I. $$\int_1^{\infty} x^2 e^{-x^3}\,dx = \lim_{t\to\infty} \int_1^t x^2 e^{-x^3}\,dx$$

$$= \lim_{t\to\infty} \left(-\tfrac{1}{3}e^{-x^3}\right)\Big]_1^t = 0 + \tfrac{1}{3}e^{-1}.$$

II. equals $\sum_{n=1}^{\infty} 1/(3n+2)$:

$$\int_1^{\infty} \frac{1}{3x+2}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{3x+2}\,dx$$

$$= \lim_{t\to\infty} \left(\tfrac{1}{3}\ln|3x+2|\right)\Big]_1^t = \infty.$$

Clicker Question 2

The "p-series"

For which real numbers p does the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

A. converges for $p < 0$, but diverges for $p \geq 0$

B. diverges for every p

C. converges for $p < 1$, but diverges for $p \geq 1$

D. converges for $p > 1$, but diverges for $p \leq 1$

E. converges for $p > 0$, but diverges for $p \leq 0$

Using the Integral Test

When $p > 0$: compare to

$$\int_1^\infty \frac{1}{x^p}\,dx = \lim_{t\to\infty} \int_1^t x^{-p}\,dx$$

$$= \lim_{t\to\infty} \left(\frac{t^{1-p}}{1-p} - \frac{1^{1-p}}{1-p} \right)$$

$$= \begin{cases} 0 - \frac{1}{1-p}, & \text{if } 1-p < 0, \\ \infty, & \text{if } 1-p > 0. \end{cases}$$

($p = 1$ is handled separately.)
When $p \leq 0$, use the Test for Divergence.