Friday, March 20

Clicker Questions

Clicker Question 1

Types of convergence

The series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(3^n)}{3^n}$ is . . .

We can rule out . . .

C and D: conv. always $\implies$ *one* of absolutely conv. or conditionally conv., but *never both*

E: absolutely conv. always $\implies$ conv.

A. . . . both absolutely convergent and convergent, but not conditionally convergent.

B. . . . both conditionally convergent and convergent, but not absolutely convergent.

C. . . . convergent, but neither absolutely convergent nor conditionally convergent.

D. . . . absolutely convergent and conditionally convergent and convergent.

E. . . . both absolutely convergent and conditionally convergent, but not convergent.

Clicker Question 2

A limit of a ratio

Suppose we have the series

$$\sum_{n=3}^{\infty} a_n\,, \quad \text{where} \quad a_n = \frac{(n-1)!}{2^{n(n-1)}}\,.$$

What is $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$?

A. ∞

B. 1

C. 0

D. $1/2$

E. none of the above

The calculation

$$\begin{aligned}
\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n\to\infty} \frac{n!/2^{(n+1)n}}{(n-1)!/2^{n(n-1)}} \\
&= \lim_{n\to\infty} \frac{n!}{(n-1)!} \frac{2^{n^2-n}}{2^{n^2+n}} \\
&= \lim_{n\to\infty} n \cdot \frac{1}{2^{(n^2+n)-(n^2-n)}} \\
&= \lim_{n\to\infty} n \cdot \frac{1}{2^{2n}} = \lim_{n\to\infty} \frac{n}{4^n} = 0.
\end{aligned}$$