Monday, March 30

Clicker Questions

Clicker Question 1

New power series from old

Find a power series representation for the function $\ln(1-x)$ on the interval $(-1, 1)$.

A. $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \cdots$

B. $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$

C. $x + 2x^2 + 3x^3 + 4x^4 + 5x^5 + \cdots$

D. $x - 2x^2 + 3x^3 - 4x^4 + 5x^5 - \cdots$

E. none of the above

Integrating a power series

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

$$-\ln|1-x| = \int \frac{1}{1-x}\,dx$$

$$= C + \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1}.$$

Plug in $x = 0$ on both sides to get $C = 0$.

Clicker Question 2

Related series

Suppose that the series $\sum_{n=3}^{\infty} c_n(-6)^n$ converges. What can we say about the radius of convergence, R, of the power series

$$\sum_{n=3}^{\infty} c_n n(n-1)x^{n-2}\,?$$

A. $R = 6$

B. $R \leq 6$

C. $R \geq 6$

D. $|R| = 6$

E. none of the above

Double derivative

Let $f(x) = \sum_{n=3}^{\infty} c_n x^n$; since the series converges at $x = -6$, its radius of convergence is at least 6. But $f''(x) = \sum_{n=3}^{\infty} c_n (x^n)'' = \sum_{n=3}^{\infty} c_n n(n-1)x^{n-2}$ has the same radius of convergence as the series for $f(x)$ itself; so $R \geq 6$.