Wednesday, March 4

Clicker Questions

Clicker Question 1

Applying the Squeeze Theorem

Calculate $\lim_{n\to\infty} \dfrac{(-1)^n + 2n + 3\cos 4n}{n}$.

A. 0
B. 1
C. 2
D. 3
E. 4

Two bounding sequences

Since $(-1)^n$ is either -1 or 1, and $3\cos 4n$ is always between -3 and 3, the limit must lie between

$$\lim_{n\to\infty} \frac{2n-4}{n} \quad \text{and} \quad \lim_{n\to\infty} \frac{2n+4}{n},$$

both of which equal 2.

Clicker Question 2

Will this problem send you to the hospital?

Evaluate $\lim_{n\to\infty} \frac{\ln n}{n^{1/9}}$.

A. converges to 9
B. diverges
C. converges to $1/9$
D. converges to 1
E. converges to 0

Using l'Hôpital's Rule

It suffices to calculate $\lim_{x\to\infty} \frac{\ln x}{x^{1/9}}$, which is an $\frac{\infty}{\infty}$ indeterminate form. Its limit is therefore equal to

$$\lim_{x\to\infty} \frac{(\ln x)'}{(x^{1/9})'} = \lim_{x\to\infty} \frac{1/x}{x^{-8/9}/9}$$
$$= \lim_{x\to\infty} \frac{9}{x^{1/9}} = 0.$$