Friday, March 6

Clicker Questions

Clicker Question 1

Partial sums that simplify

By finding a formula for s_n, the nth partial sum of the series

$$\sum_{i=1}^{\infty} \left(2^{3/i} - 2^{3/(i+1)}\right),$$

calculate the value that the series converges to.

A. 7

B. 0

C. 8

D. -1

E. none of the above

A telescoping sum

$$s_n = \left(2^{3/1} - 2^{3/2}\right) + \left(2^{3/2} - 2^{3/3}\right) + \cdots + \left(2^{3/(n-1)} - 2^{3/n}\right) + \left(2^{3/n} - 2^{3/(n+1)}\right)$$

$$= 2^{3/1} - 2^{3/(n+1)}.$$ So

$$\sum_{i=1}^{\infty} \left(2^{3/i} - 2^{3/(i+1)}\right) = \lim_{n\to\infty} \left(2^{3/1} - 2^{3/(n+1)}\right) = 8 - 2^0 = 7.$$

Clicker Question 2

Dividing a complicated series into simpler ones

Using previous examples from this lecture, calculate

$$\sum_{i=1}^{\infty} \left(\frac{3}{7} \left(2^{3/i} - 2^{3/(i+1)} \right) - \frac{4}{i^2 + 8i + 15} \right).$$

A. -2

B. 6.1

C. 2.1

D. -6

E. none of the above

Series laws in action

$$\begin{aligned}
&\sum_{i=1}^{\infty} \left(\frac{3}{7} \left(2^{3/i} - 2^{3/(i+1)} \right) - \frac{4}{i^2 + 8i + 15} \right) \\
&= \frac{3}{7} \sum_{i=1}^{\infty} \left(2^{3/i} - 2^{3/(i+1)} \right) \\
&\qquad - \frac{4}{10} \sum_{i=1}^{\infty} \frac{10}{i^2 + 8i + 15} \\
&= \frac{3}{7} \cdot 7 - \frac{4}{10} \cdot \frac{9}{4} = 3 - 0.9.
\end{aligned}$$

Clicker Question 3

Almost a geometric series

Calculate $.7\overline{90} = .790909090\ldots$

$$= \frac{7}{10} + \frac{9}{100} + \frac{9}{10{,}000} + \frac{9}{1{,}000{,}000} + \frac{9}{100{,}000{,}000} + \cdots.$$

A. $\frac{4}{5}$

B. $\frac{1}{11}$

C. $\frac{87}{110}$

D. $\frac{70}{99}$

E. none of the above

From our formula

$$\frac{7}{10} + \sum_{i=1}^{\infty} \frac{9}{100}\left(\frac{1}{100}\right)^{r-1}$$

$$= \frac{7}{10} + \frac{9/100}{1 - 1/100}$$

$$= \frac{7}{10} + \frac{9}{100 - 1}$$

$$= \frac{7}{10} + \frac{1}{11}.$$

Clicker Question 4

Staring a series at a different index

Let $\{a_n\}$ be a sequence. Suppose you know that the series $\sum_{n=1}^{\infty} a_n = S$ converges to some value S. What can you say about the series $\sum_{n=4}^{\infty} a_n$?

A. it converges only if $a_1 > a_2 > a_3 > a_4$

B. it always converges, to $S - (a_1 + a_2 + a_3)$

C. it always diverges

D. it converges only if a_1, a_2, a_3 are positive

E. there's not enough information to tell

Two related sums

The partial sums of

$$\sum_{n=4}^{\infty} a_n = a_4 + a_5 + \cdots$$

are all $(a_1 + a_2 + a_3)$ less than the partial sums of

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + a_4 + a_5 + \cdots .$$