

## Math 220, Section 201

### Homework #1

due Friday, January 18, 2002 at the beginning of class

#### Warm-Up Questions—do not hand in

- I. Lay, p. 9, #1.9
- II. Lay, p. 14, #2.5
- III. We define the term *strictly increasing* as follows. A function  $f$  is strictly increasing if and only if: for every  $x$  and for every  $y$ , if  $x > y$ , then  $f(x) > f(y)$ . Write what it means if you are told that the function  $g$  is not strictly increasing.
- IV. We define the term *surjective* as follows. A function  $f$  is surjective if and only if: for every  $y$ , there exists an  $x$  such that  $f(x) = y$ . Write what it means if you are told that the function  $g$  is not surjective.
- V. Lay, p. 29, #4.5

**January 18's quiz** will be one of these five warm-up questions.

#### Homework Questions—hand these in

- I. Each of the following bits of logical reasoning is an example of one of the tautologies (a)–(q) in Example 3.12 of Lay (p. 20). For each statement, write down which of those tautologies it represents. (It might help to mentally rephrase parts of the statements in “if-then” form.)
  - (a) It's sunny, and I smile whenever it's sunny; so I'm smiling!
  - (b) Every Monday Joe eats pizza, and every Friday Joe eats fishsticks. Today is either Monday or Friday, therefore Joe is eating either pizza or fishsticks.
  - (c) Widgets and gromits are exactly the same thing, since all widgets are gromits, and everything that's not a widget is also not a gromit.
  - (d) Becky looks unhappy. If she had gotten the promotion, she wouldn't look unhappy. She must not have gotten the promotion.
- II. Verify, using truth tables, that  $(p \wedge \sim q) \Rightarrow \sim r$  and  $(p \wedge r) \Rightarrow q$  mean the same thing. Say what this implies about the statement
$$((p \wedge \sim q) \Rightarrow \sim r) \Leftrightarrow ((p \wedge r) \Rightarrow q).$$
- III. Translate the statement “If  $x$  is rational and  $y$  is not rational, then  $x + y$  is not rational” using the equivalence given in the previous problem. Use this translation to prove the statement. (A number is rational if and only if it can be written as  $a/b$ , where  $a$  and  $b$  are integers and  $b \neq 0$ .)

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- IV. (a) Prove: If  $x - 1 > 0$ , then  $x/(x - 1) \leq 2$  implies  $x < 1$  or  $x \geq 2$ .  
(b) Prove: If  $x - 1 = 0$ , then  $x/(x - 1) \leq 2$  implies  $x < 1$  or  $x \geq 2$ .  
(c) Prove: If  $x - 1 < 0$ , then  $x/(x - 1) \leq 2$  implies  $x < 1$  or  $x \geq 2$ .  
(d) You will probably agree that your work done in solving the first three parts of this problem established the following fact:

$$\text{if } x/(x - 1) \leq 2, \text{ then } x < 1 \text{ or } x \geq 2.$$

Explain what this has to do with the tautology

$$((p \vee q \vee r) \wedge (p \Rightarrow s) \wedge (q \Rightarrow s) \wedge (r \Rightarrow s)) \Rightarrow s.$$

(You should convince yourself that this is indeed a tautology, but you don't have to prove it.)