Math 220, Section 201
Homework #2
due Friday, January 25, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, p. 42, #5.8

II. Let A be a set. Prove that $\emptyset \subseteq A$.

III. Show that $A \subset B$ if and only if $A \subseteq B$ and there exists an element $x \in B$ such that $x \notin A$.

IV. Let A and B be subsets of $\mathbb{R}$. Show that $\{x \in \mathbb{R} \colon \sim(x \in A \Rightarrow x \in B)\} = A \setminus B$.

V. Lay, p. 92, #10.6

January 25's quiz will be one of these five warm-up questions.

Homework Questions—hand these in

I. Lay, p. 42, #5.4. Then, find the intersection of each of the sets A, B, and C with the half-open interval $[2, 6)$.

II. Suppose that S and T are sets with $S \subseteq T$. Match each of the three expressions on the left with the set on the right that it equals. Prove your answers.

1. $S \cap T$	a. $\emptyset$
2. $S \setminus T$	b. S
3. $S \cup T$	c. T

III. Lay, p. 92, #10.16

IV. Consider the following calculations:

$$8\left(1+\tfrac{2}{1}\right) = 24 = 5^2 - 1$$
$$8\left(1+\tfrac{2}{1}\right)\left(1+\tfrac{2}{2}\right) = 48 = 7^2 - 1$$
$$8\left(1+\tfrac{2}{1}\right)\left(1+\tfrac{2}{2}\right)\left(1+\tfrac{2}{3}\right) = 80 = 9^2 - 1$$
$$8\left(1+\tfrac{2}{1}\right)\left(1+\tfrac{2}{2}\right)\left(1+\tfrac{2}{3}\right)\left(1+\tfrac{2}{4}\right) = 120 = 11^2 - 1$$

From this pattern, make a conjecture as to the value of

$$8\left(1+\tfrac{2}{1}\right)\left(1+\tfrac{2}{2}\right) \times \cdots \times \left(1+\tfrac{2}{n}\right)$$

for any natural number n, and then prove your conjecture by induction.