Math 220, Section 201
Homework #5
due Friday, March 1, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Lay, p. 146, #16.1(a),(c)

II. Lay, p. 146, #16.2(a),(b)

III. Lay, p. 146, #16.4(c)

IV. Lay, p. 147, #16.5(d)

V. Lay, p. 147, #16.6(b)

March 1's quiz will be one of these five warm-up questions.

Homework Questions—hand these in

I. (a) Let S be a subset of $\mathbb{R}$, and suppose that x is a boundary point of S that is not in S. For each $n \in \mathbb{N}$, define $U_n = (-\infty, x - \frac{1}{n}) \cup (x + \frac{1}{n}, \infty)$. Prove that $S \subseteq \bigcup_{n=1}^{\infty} U_n$ but that $S \not\subseteq \bigcup_{n=1}^{k} U_n$ for every $k \in \mathbb{N}$. (Hint: first find simpler expressions for $\bigcup_{n=1}^{\infty} U_n$ and $\bigcup_{n=1}^{k} U_n$.)

(b) Prove that every compact subset of $\mathbb{R}$ is closed. Do not assume the Heine-Borel Theorem. (But do use part (a) of this problem!)

II. Lay, p. 146, #16.4(b),(e)

III. Lay, p. 147, #16.6(a),(c)

IV. You only need to do *one* of the following two problems. (If you want to do them both, you can get extra credit!)

(a) Suppose that A is a subset of $\mathbb{R}$ such that no rational number is an accumulation point of A. Prove that $\operatorname{int} A = \emptyset$.

(b) Suppose that B is a subset of $\mathbb{R}$ such that every rational number is an accumulation point of B. Prove that $\operatorname{cl} B = \mathbb{R}$.