Math 220, Section 201
Homework #7

due Friday, March 15, 2002 at the beginning of class

Warm-Up Questions—do not hand in

I. Finish the proof of Theorem 18.3 by proving that a bounded decreasing sequence is convergent.

II. Lay, p. 161, #18.8

III. Lay, p. 162, #18.9

IV. Let (s_n) be a sequence of nonnegative numbers converging to L. Suppose that the sequence $(\sqrt{s_n})$ is also convergent. Prove that $\lim \sqrt{s_n} = \sqrt{L}$. (Hint: you can do this one without epsilons.)

V. Suppose that a sequence (s_n) diverges to $-\infty$. Show that every subsequence of (s_n) also diverges to $-\infty$.

March 15's quiz will be one of these five warm-up questions.

Homework Questions—hand these in

I. Lay, p. 161, #18.4

II. Lay, p. 162, #18.11

III. Define a sequence (s_n) by $s_1 = 1$ and $s_{n+1} = \frac{2}{3}(s_n + \sqrt{s_n})$.
   (a) Prove that (s_n) is increasing, bounded below by 0, and bounded above by 4.
   (b) Prove (using part (a) if you wish) that (s_n) is convergent, and calculate $\lim s_n$.
   (c) Which of the above properties would change if $s_1 = 3$ rather than $s_1 = 1$?

IV. Lay, p. 169, #19.6

V. We basically know four possible behaviors for sequences: convergent; divergent to $+\infty$; divergent to $-\infty$; and divergent but not to $\pm\infty$. Give an example of a single sequence (s_n) that has subsequences with each of these four behaviors (indicate the appropriate subsequences).